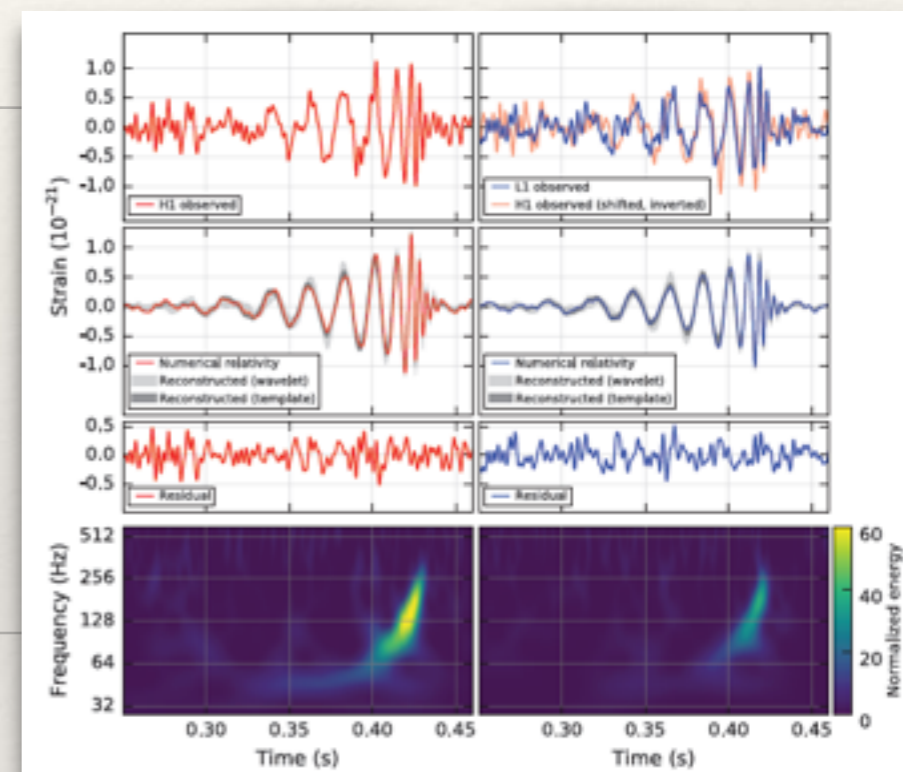




*Stas Babak (for LIGO and VIRGO collaboration).
Albert Einstein Institute (Potsdam-Golm)*

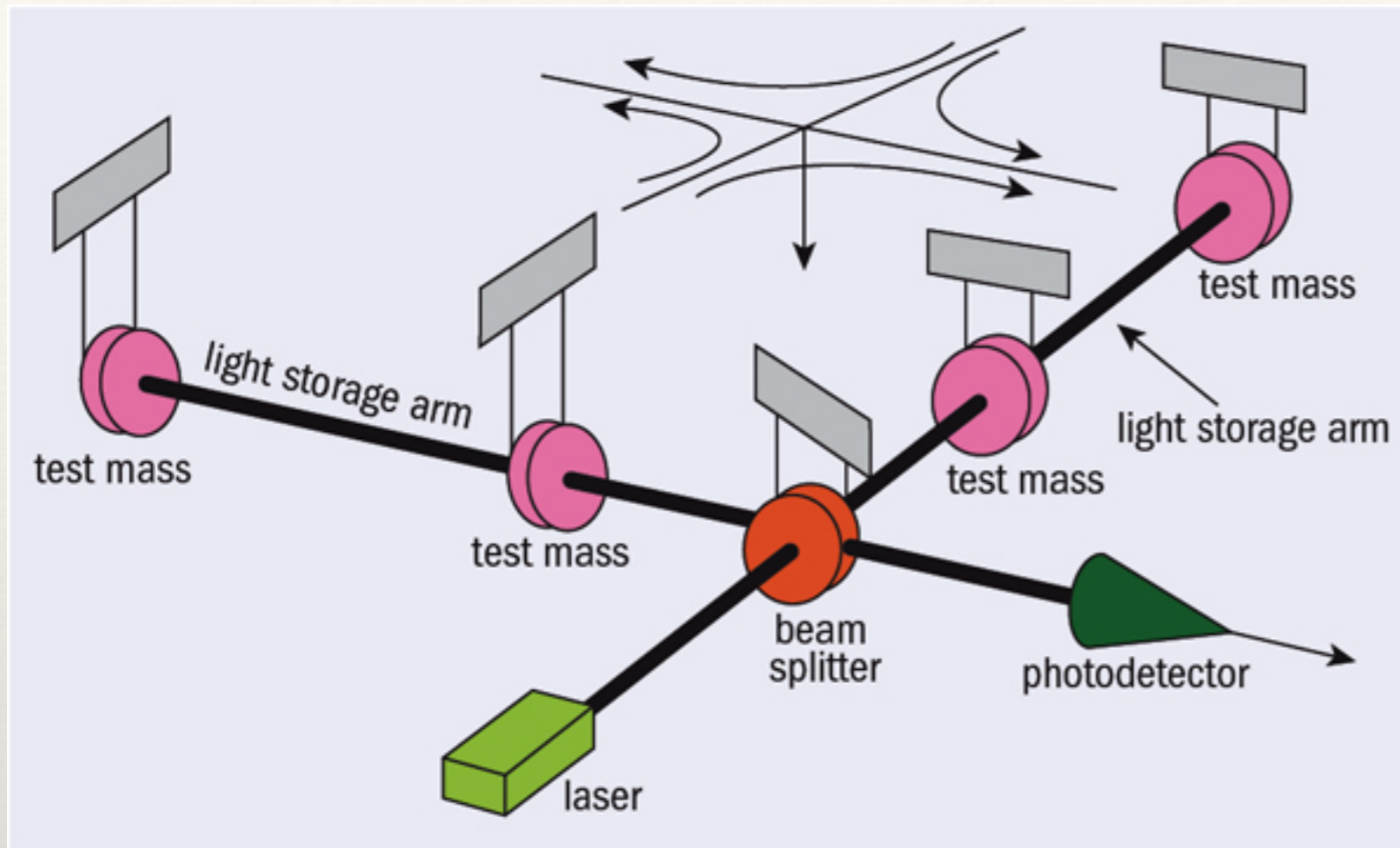
What have we learned from coalescing Black Hole binary GW150914

LIGO_DCC:G1600346



PRL 116, 061102 (2016)

Principles of GW detection

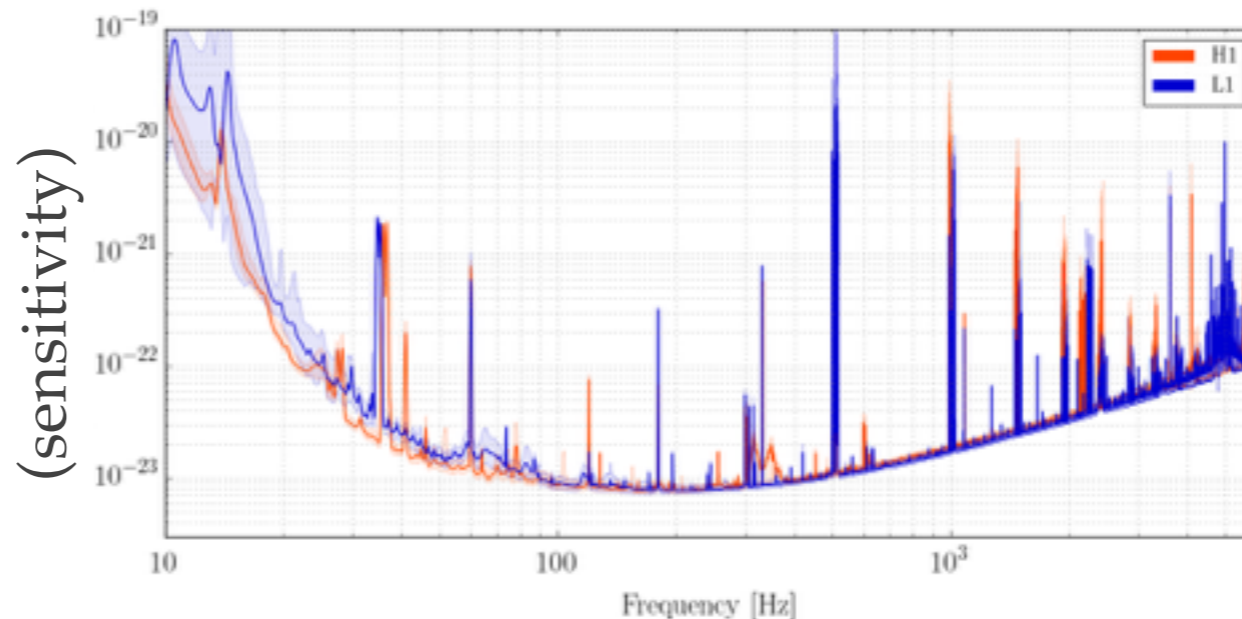


$$\Delta L = \delta L_x - \delta L_y = h(t)L$$

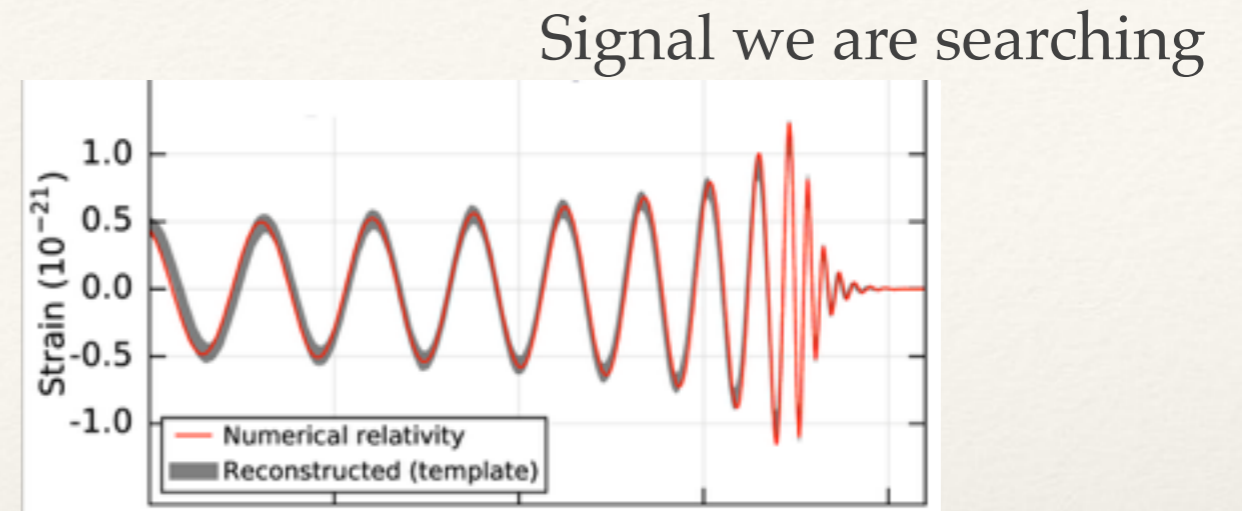
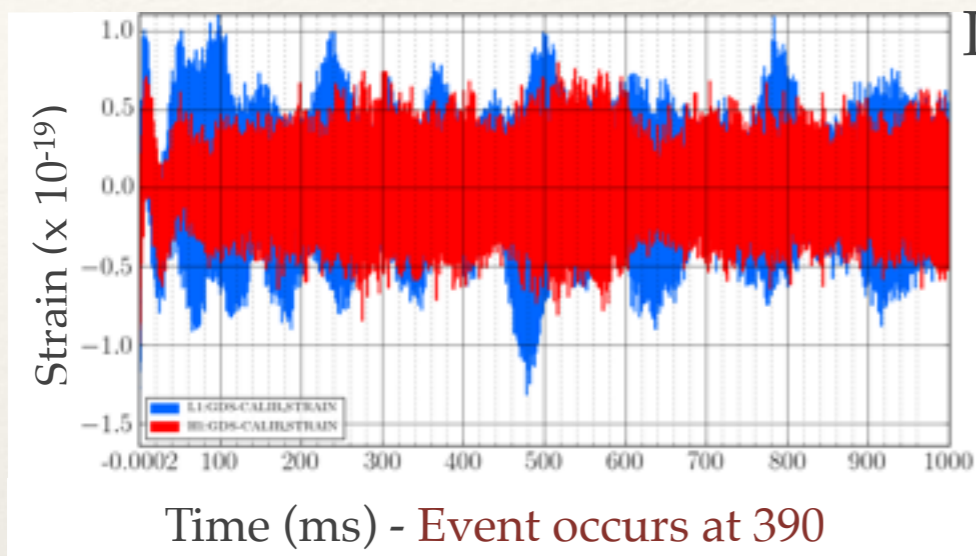
GW strain

We measure difference in the proper distance between beam splitter and end mirrors using laser interferometry

Amplitude spectral density (sensitivity)



Matched filtering



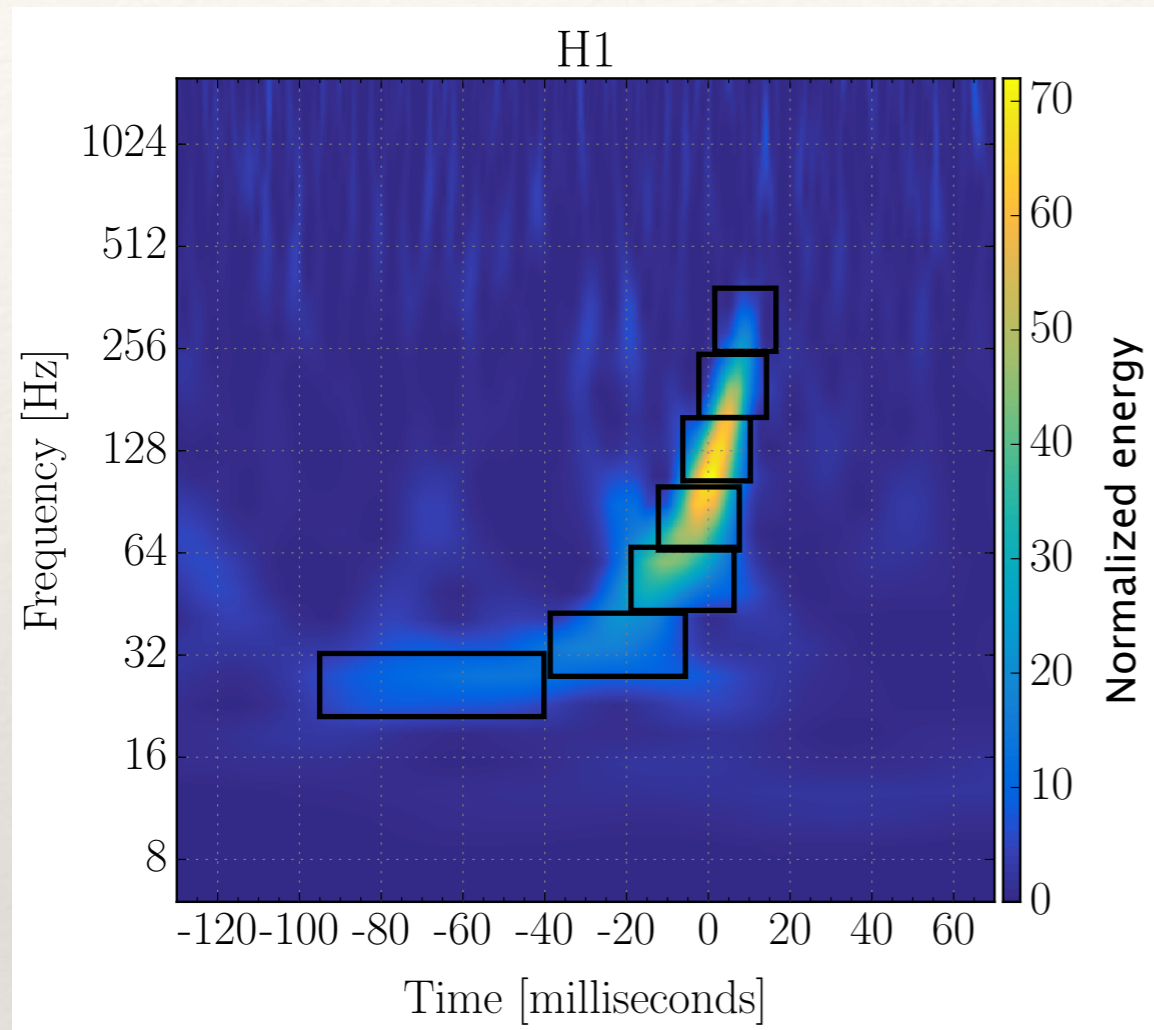
We employ matched filtering: searching the data (deep inside the noise) using template waveform. This implies that we need very accurate model of the signal (to control systematic errors and loss in the detection).

$$\rho = \int_0^\infty \frac{\tilde{d}(f)\tilde{h}^*(f)}{S(f)} \quad \text{Signal-to-noise ratio}$$

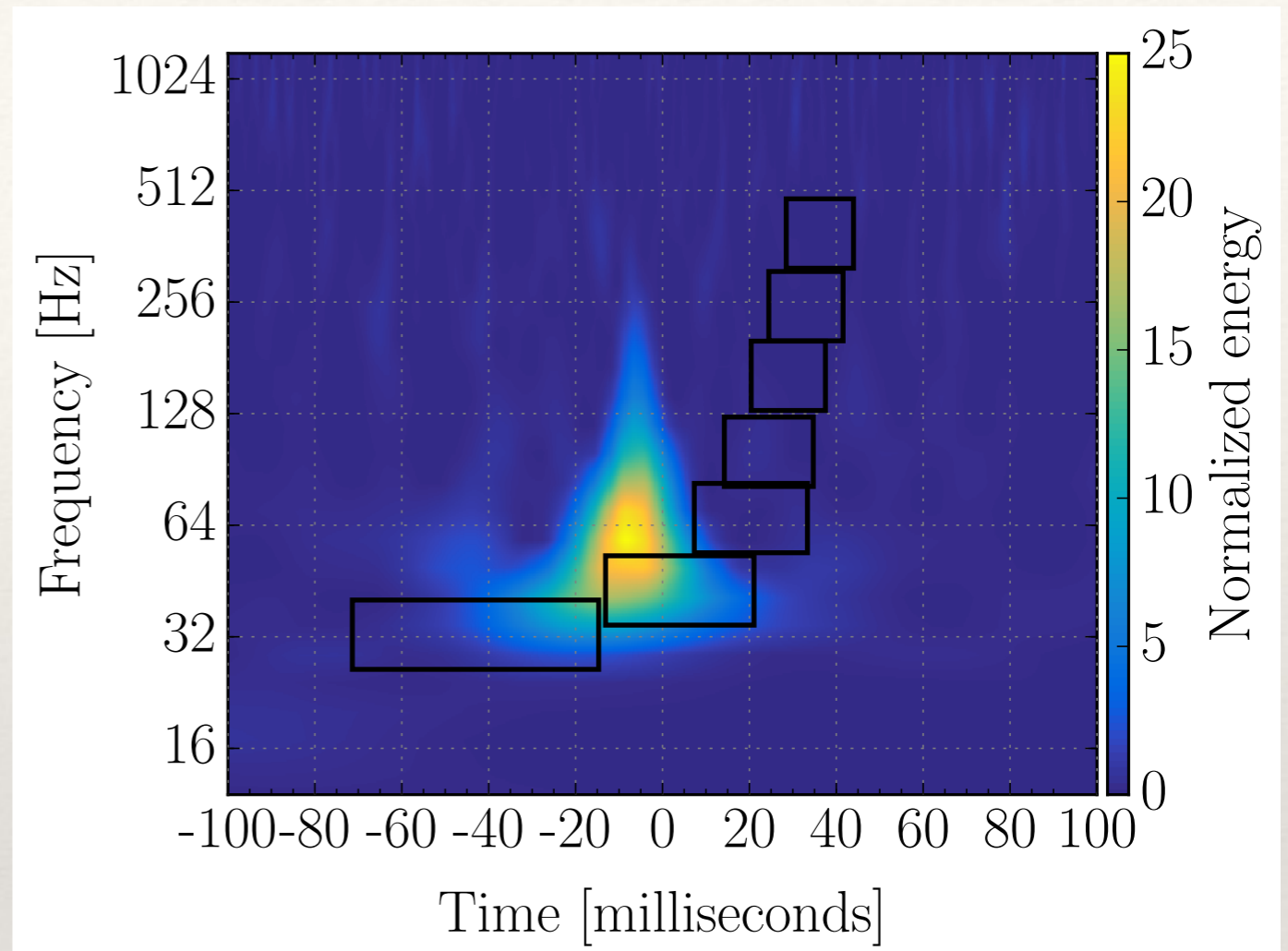
$$\mathcal{L}(\vec{d}|\vec{\vartheta}) \propto \exp \left[\frac{1}{2} \sum_{k=1,2} \left\langle h_k(\vec{\vartheta}) - d_k | h_k(\vec{\vartheta}) - d_k \right\rangle \right] \quad \text{Likelihood}$$

Consistency check

Real signal

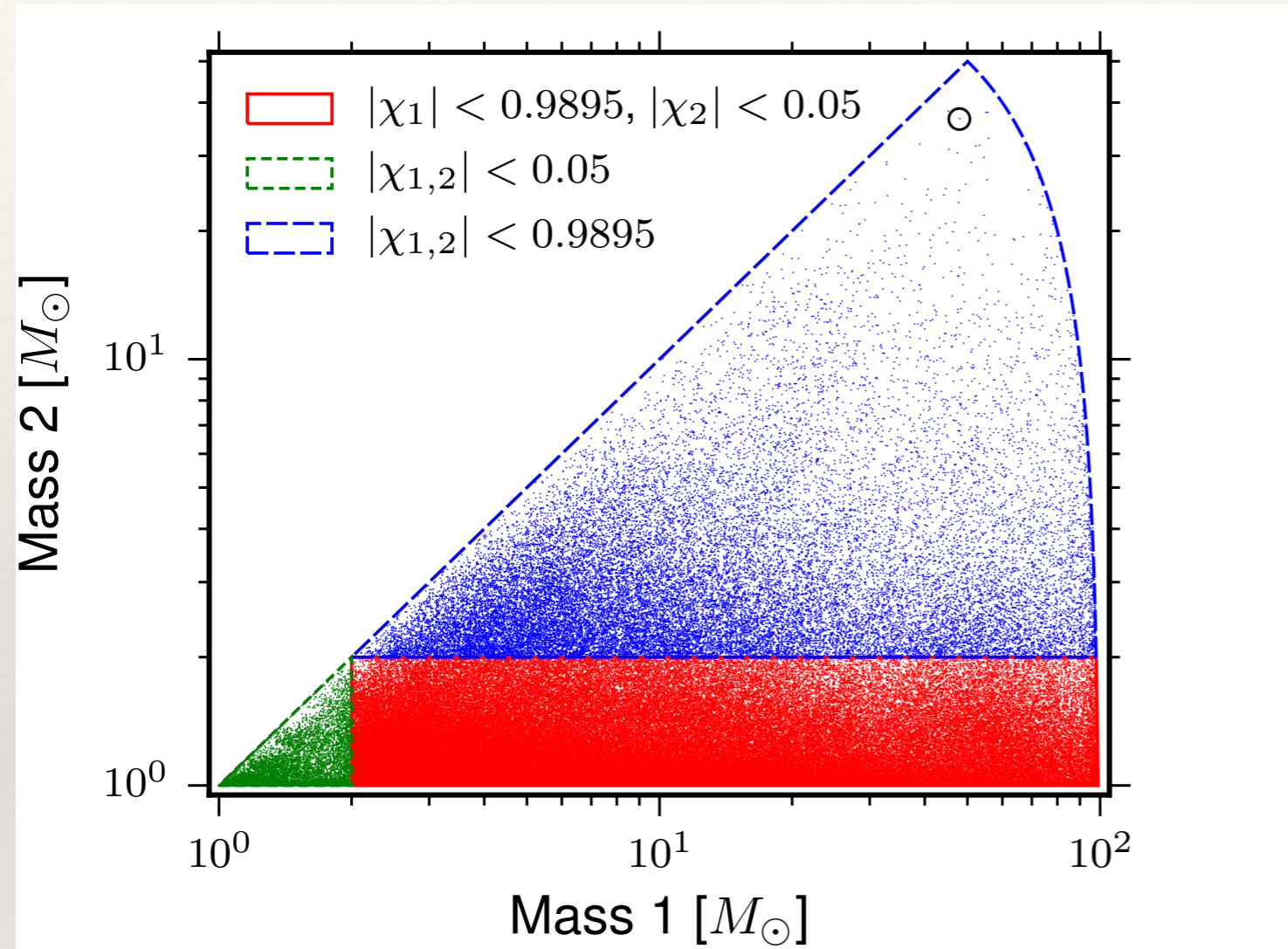


Instrumental artifact



The noise is not Gaussian: need to introduce additional consistency checks into the detection statistic (distribution of power in the signal across the time/frequency).

Template bank

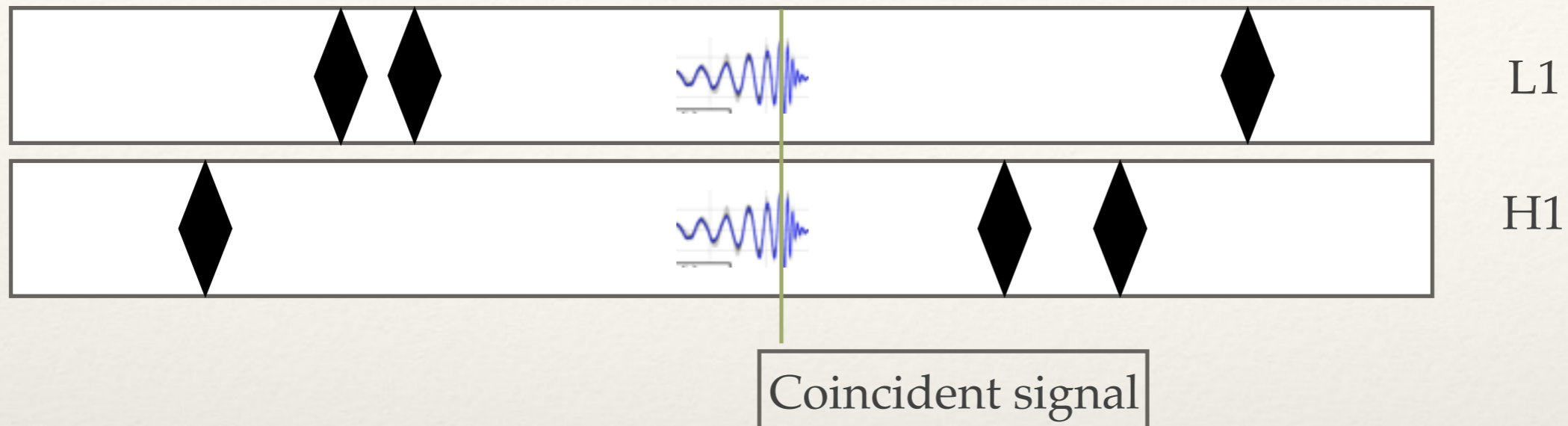


- We don't know a priori parameters of the system
- We construct the bank of templates: we populate the parameter space: uniform taking into accounts the correlation between templates ("volume of each template")
- We filter the data through each template to see which fits the best
- We have used SEOBNR (non-precessing templates)
- Total number of templates used $\sim 250,000$

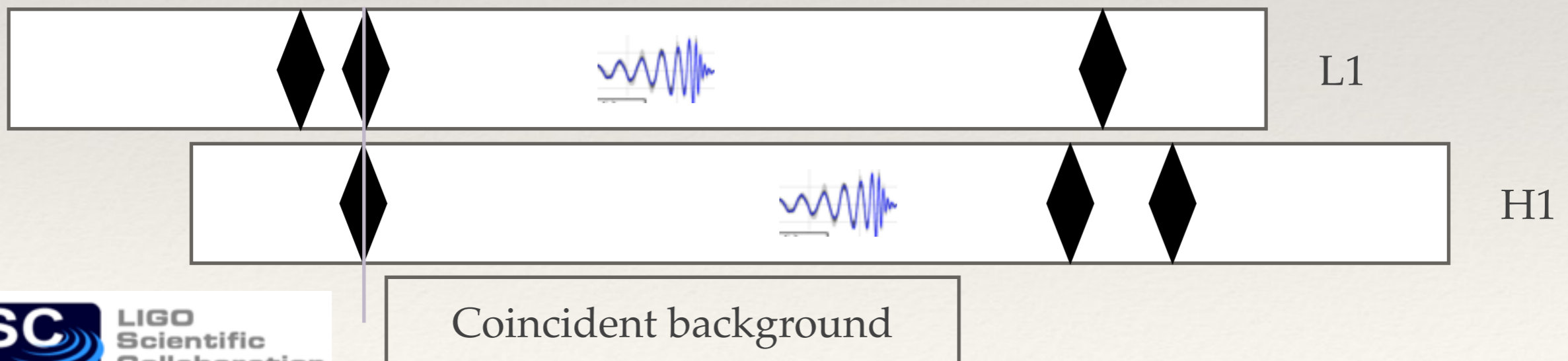
LVC: arXiv:1602.03839

Significance estimation

“Zero lag”

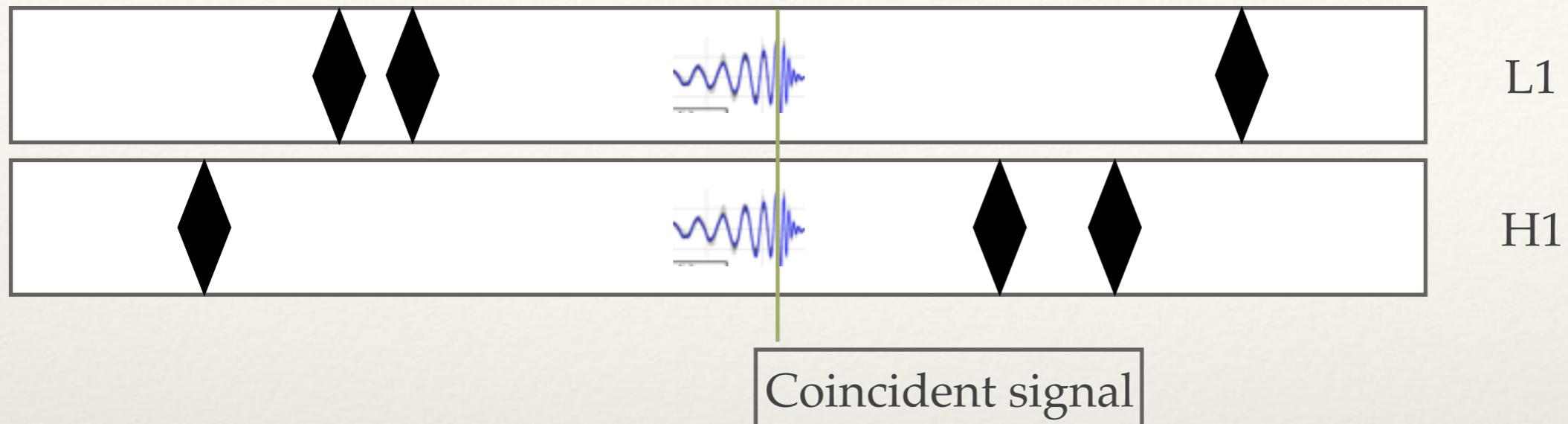


Time shift $>$ light travel time

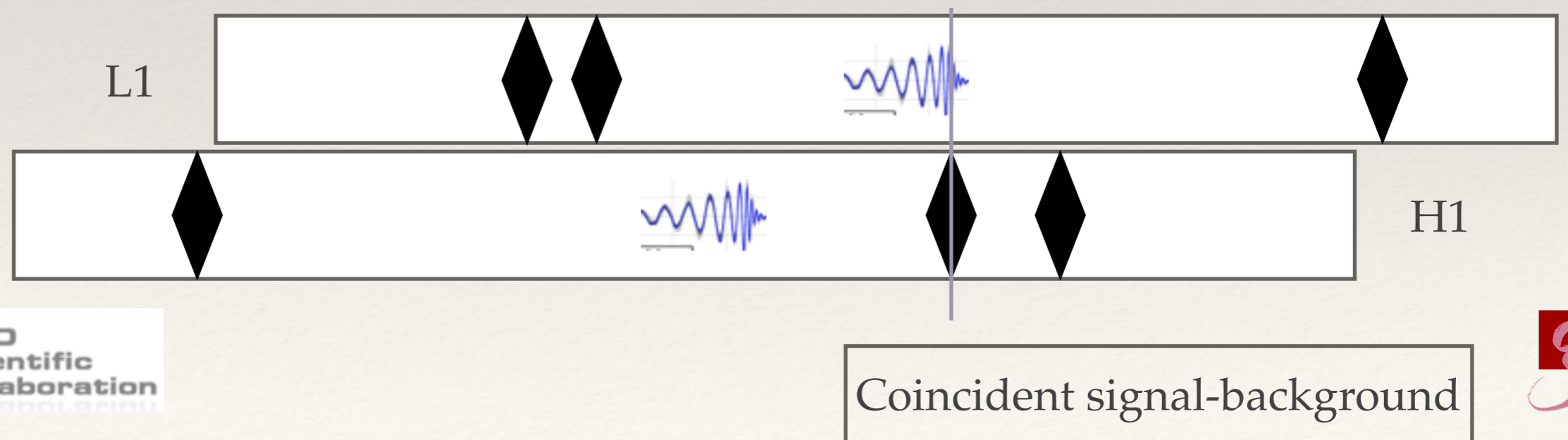


Significance estimation

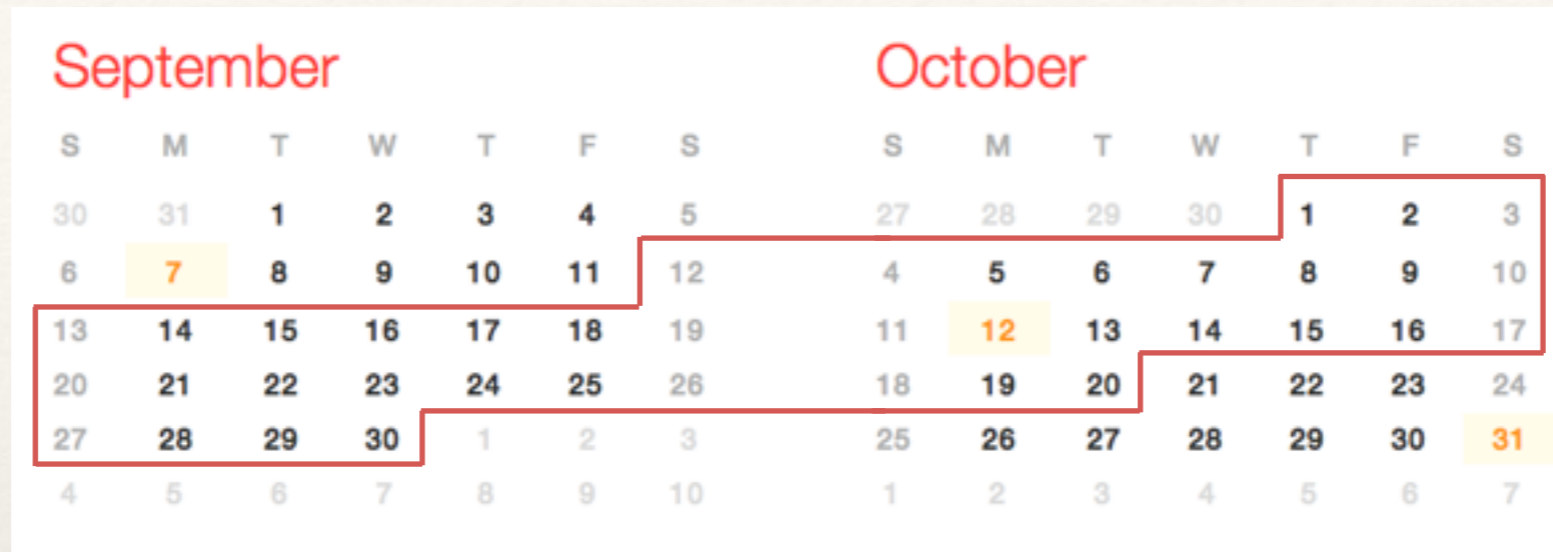
“Zero lag”



Time shift $>$ light travel time

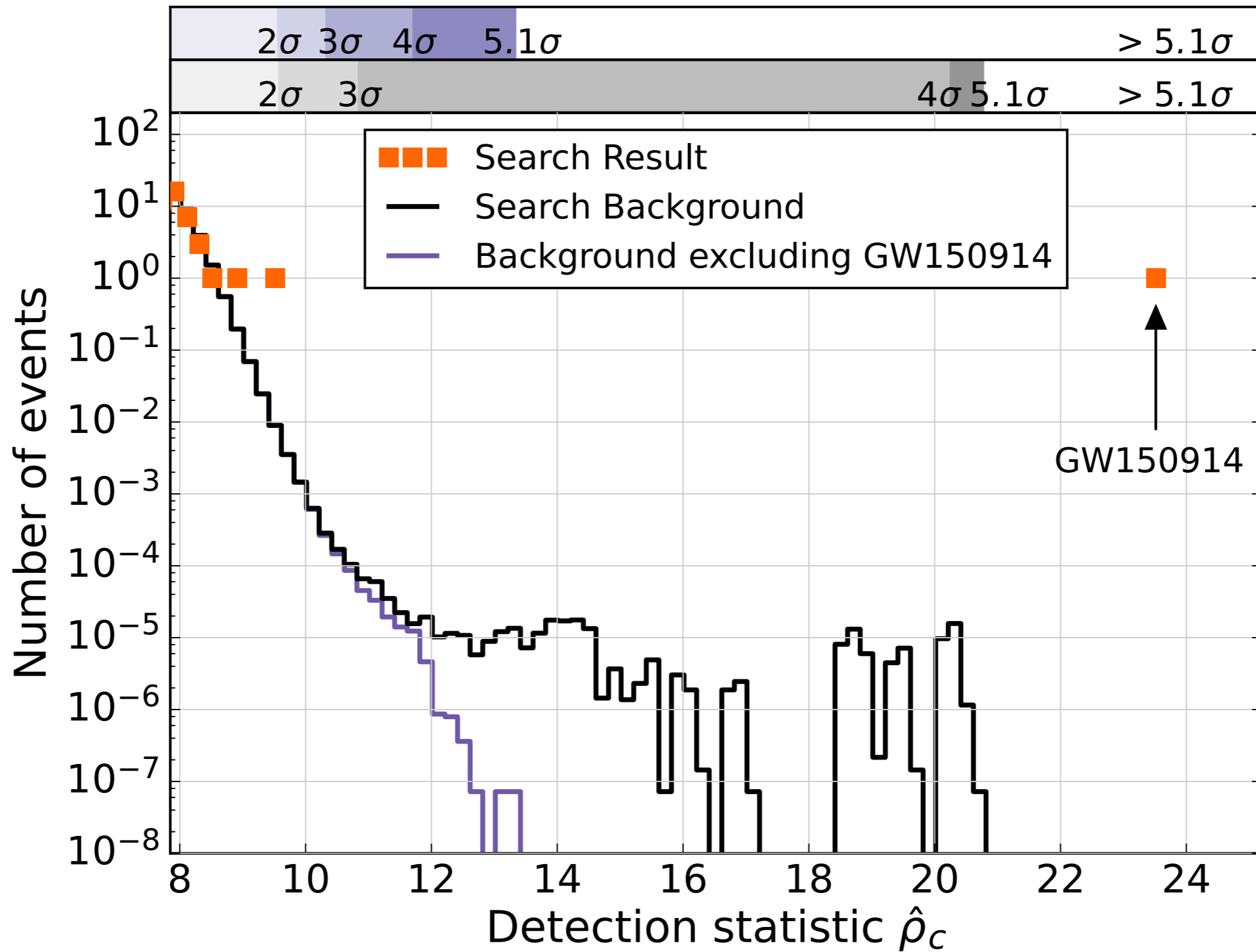


Observation



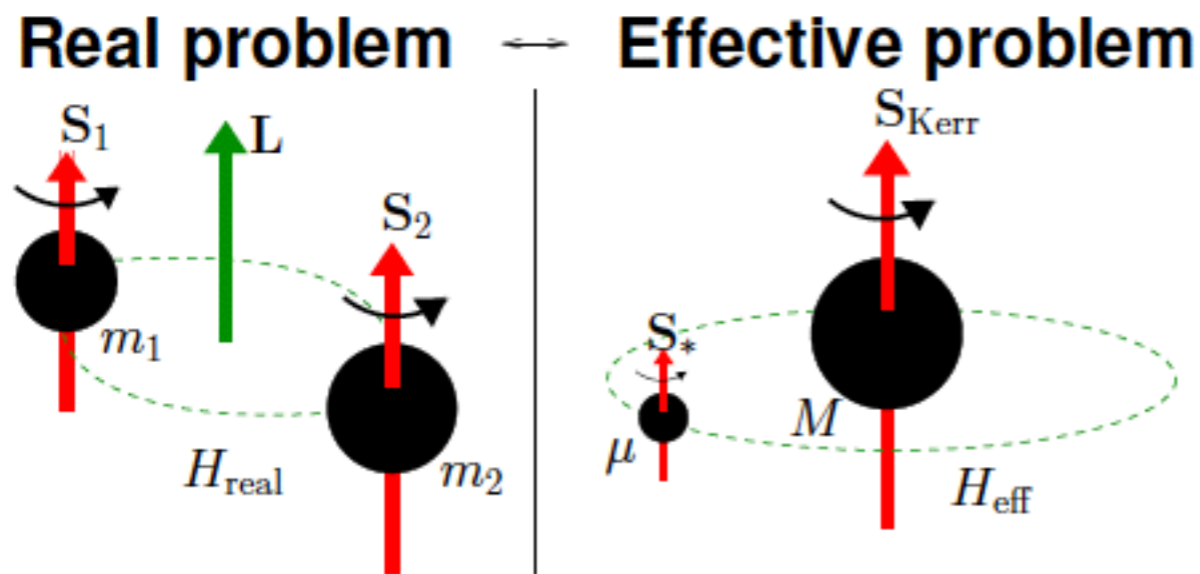
- Used 38.6 days of calendar data, which gives 18.4 days of coincident data (coincident lifetime $\sim 48\%$)
- 20.7 hours of this data were contaminated by known instrumental issues - left 17.5 days of data

Statistical significance



Signal modelling (EOB)

- **Effective-one-body (EOB) model** [Buonanno & Damour 99] describes the GR 2-body problem via
 - effective 1-body Hamiltonian (**spinning particle** in **deformed Kerr**)
 - radiation-reaction force
 - analytic inspiral-merger-ringdown waveforms $h_{\ell m}$



$$\begin{aligned}
 M &= m_1 + m_2 \\
 \mathbf{S}_{\text{Kerr}} &= \mathbf{S}_1 + \mathbf{S}_2 \\
 \mu &= \frac{m_1 m_2}{m_1 + m_2} \\
 \mathbf{S}_* &= \mathbf{S}_*(\mathbf{S}_1, \mathbf{S}_2)
 \end{aligned}$$

- Each ingredient is a **resummation of PN expressions**
- Deformation parameter: $\nu = \mu/M \in [0, 1/4]$
- **Test-particle limit** included by construction

Signal modelling (EOB)

$$H_{\text{real}} = Mc^2 \sqrt{1 + 2v \left(\frac{H_{\text{eff}}}{\mu c^2} - 1 \right)} - Mc^2$$

- **Nonspinning case:** particle in deformation of Schwarzschild [Buonanno & Damour 99]

$$H_{\text{eff}} = \mu c^2 \sqrt{A(R) \left[1 + \frac{\mathbf{p}^2}{\mu^2 c^2} + \frac{1}{\mu^2 c^2} \left(\frac{A(R)}{D(R)} - 1 \right) \left(\frac{\mathbf{R} \cdot \mathbf{p}}{R} \right)^2 \right]}$$

$$A = \underbrace{1 - 2u}_{\text{Schwarzschild}} + 2\nu u^3 + \left(\frac{94}{3} - \frac{42}{32} \pi^2 \right) \nu u^4 + \mathcal{O}(u^5) \quad (u = GM/Rc^2)$$

- **Spinning case:** spinning particle in deformation of Kerr [Barausse & Buonanno 10, 11]. *Spin-orbit* effects up to 3.5PN, *spin-spin* effects up to 2PN

Signal modelling (EOB)

- **Radiation-reaction force** [Buonanno+ 00]

$$\mathcal{F}_i \propto \frac{dE}{dt}$$

- **Waveforms** $h_{\ell m}$

- *Inspiral-plunge*: **factorized** resummation of PN $h_{\ell m}$ [Damour+ 07, 09, Pan+ 11]:

$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\text{N}} S_{\ell+m} T_{\ell m}(\rho_{\ell m})^{\ell} e^{i\delta_{\ell m}}$$

- *Ringdown*: sum of **quasinormal modes** [Kokkotas+ 99] of the remnant BH [Buonanno & Damour 00]

$$h_{\ell m}^{\text{RD}} = \sum_n A_{\ell mn} \underbrace{e^{-i\omega_{\ell mn}t}}_{\text{oscillatory}} \underbrace{e^{-t/\tau_{\ell mn}}}_{\text{damping}}$$

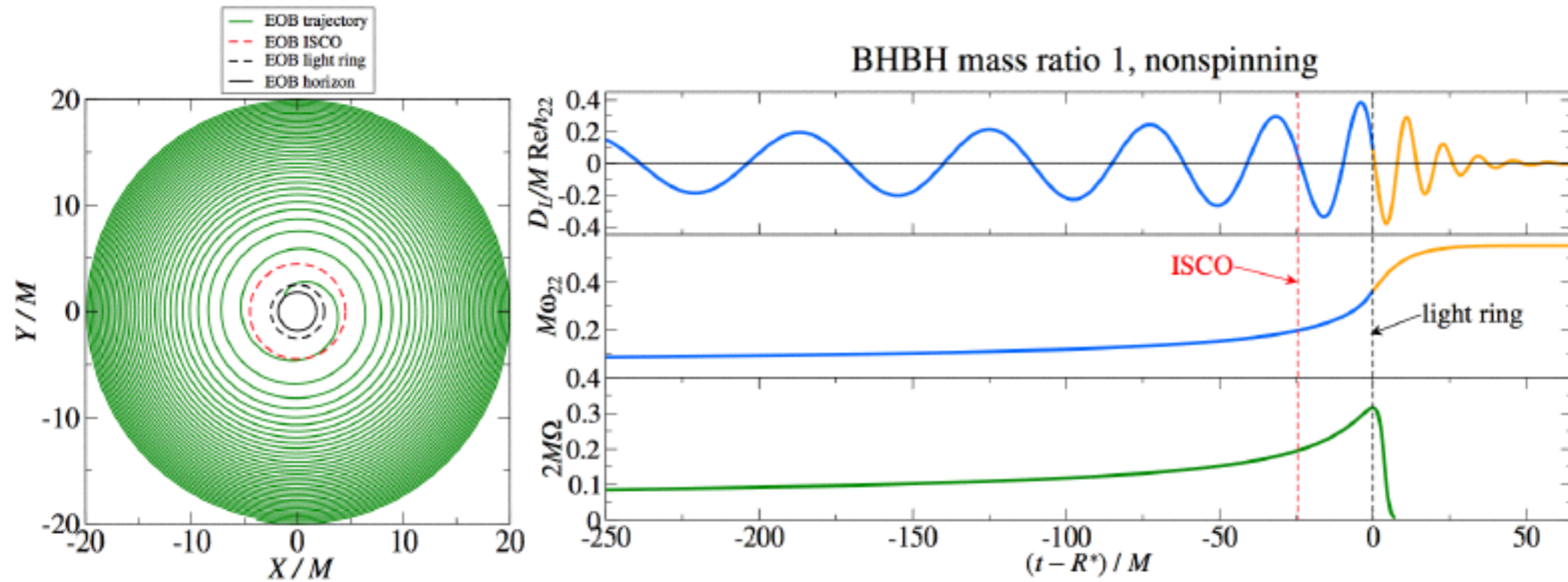
- **Nonadiabatic EOB inspiral-plunge trajectory** from Hamilton's equations

$$\frac{d\mathbf{R}}{dt} = \{\mathbf{R}, H_{\text{real}}\} \quad \frac{d\mathbf{P}}{dt} = \{\mathbf{P}, H_{\text{real}}\} + \mathcal{F}$$

- Integrate numerically from **quasicircular initial conditions**

Signal Modelling (EOB)

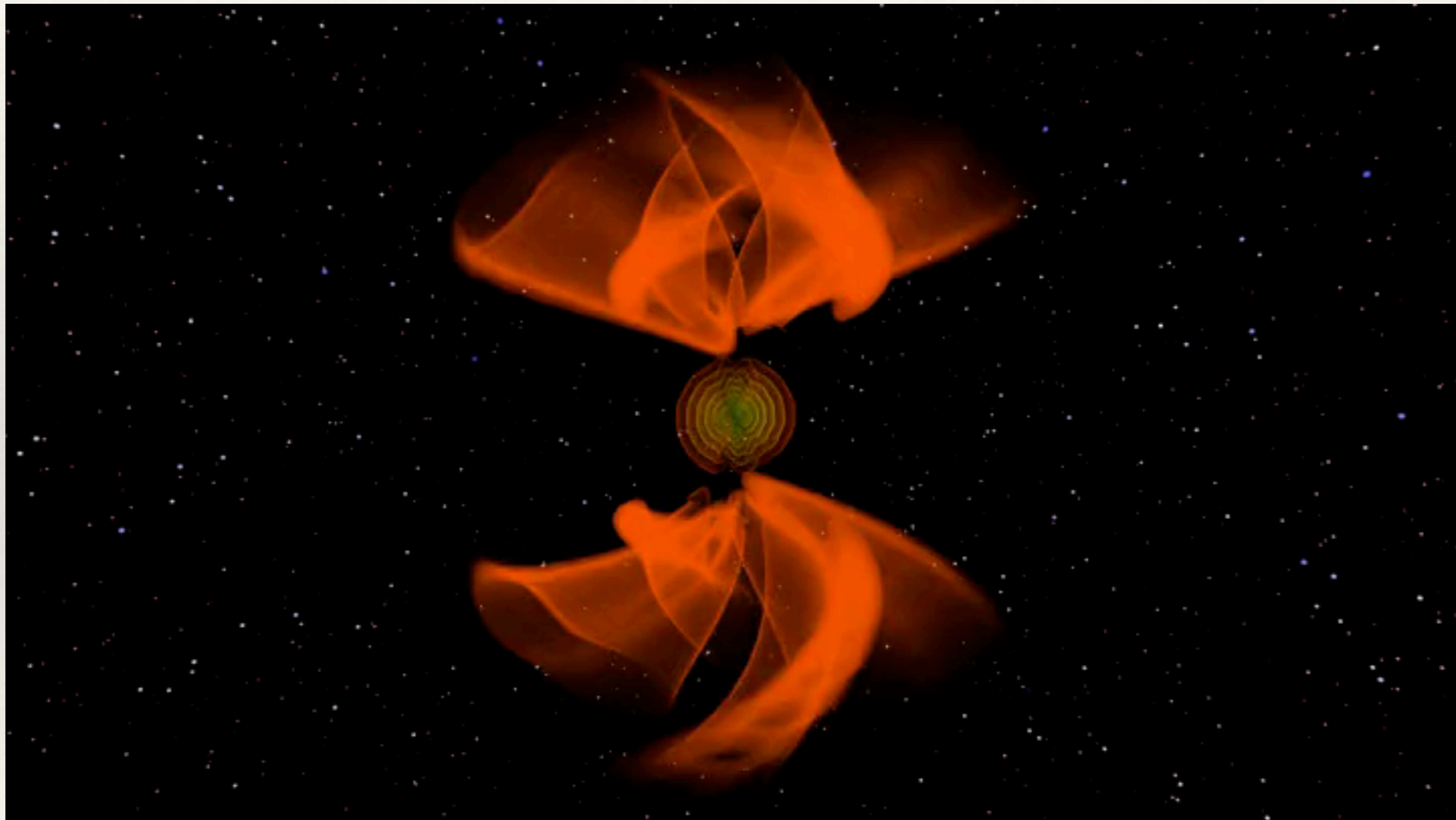
Example of constructing full signal in EOB model



- Identify RD (ringdown) attachment time based on the dynamics: **light ring**
- Use a time window near the light ring for **continuous matching RD to the inspiral-merger**
- Allow for QNM mixing in EOB if orbits become retrograde [SB, Taracchini & Buonanno (in prep)]

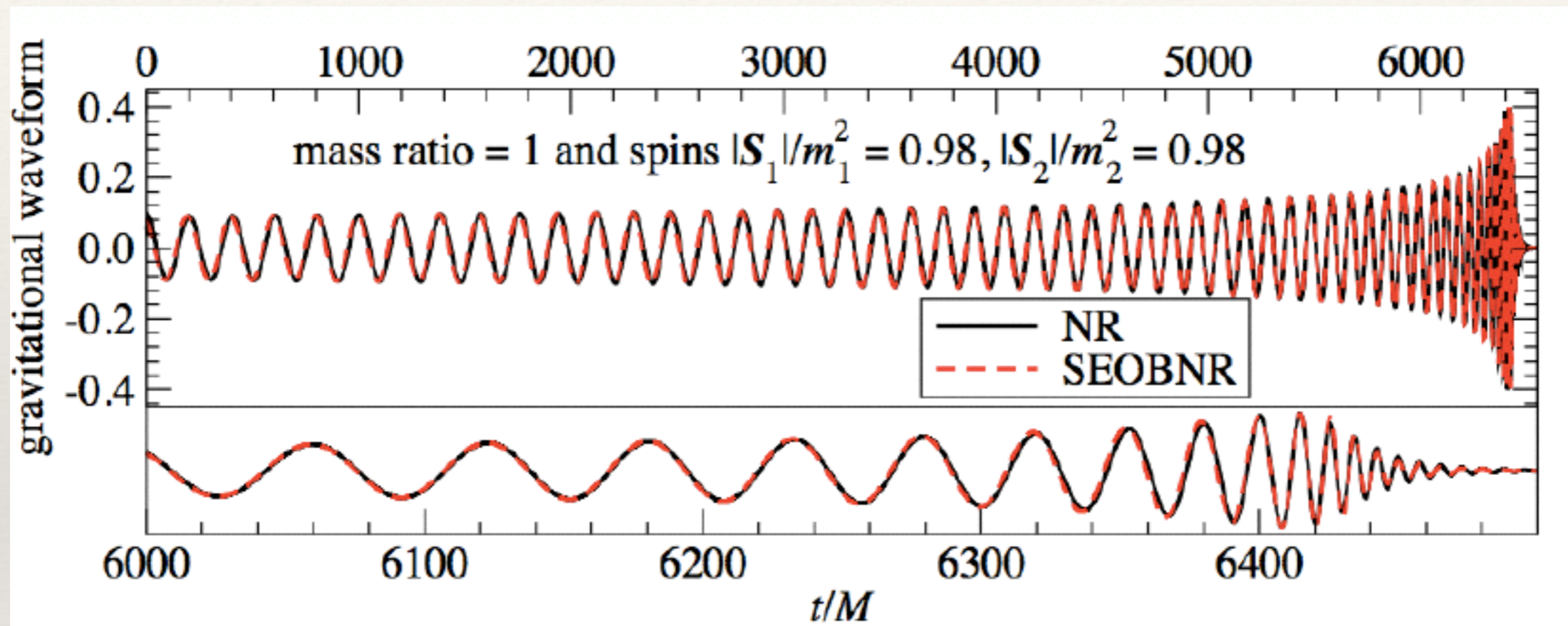
Numerical Relativity

Solving Einstein equations “exactly” numerically: computationally very demanding
rather limited number of waveforms can be generated and they are short



EOB - NR comparison

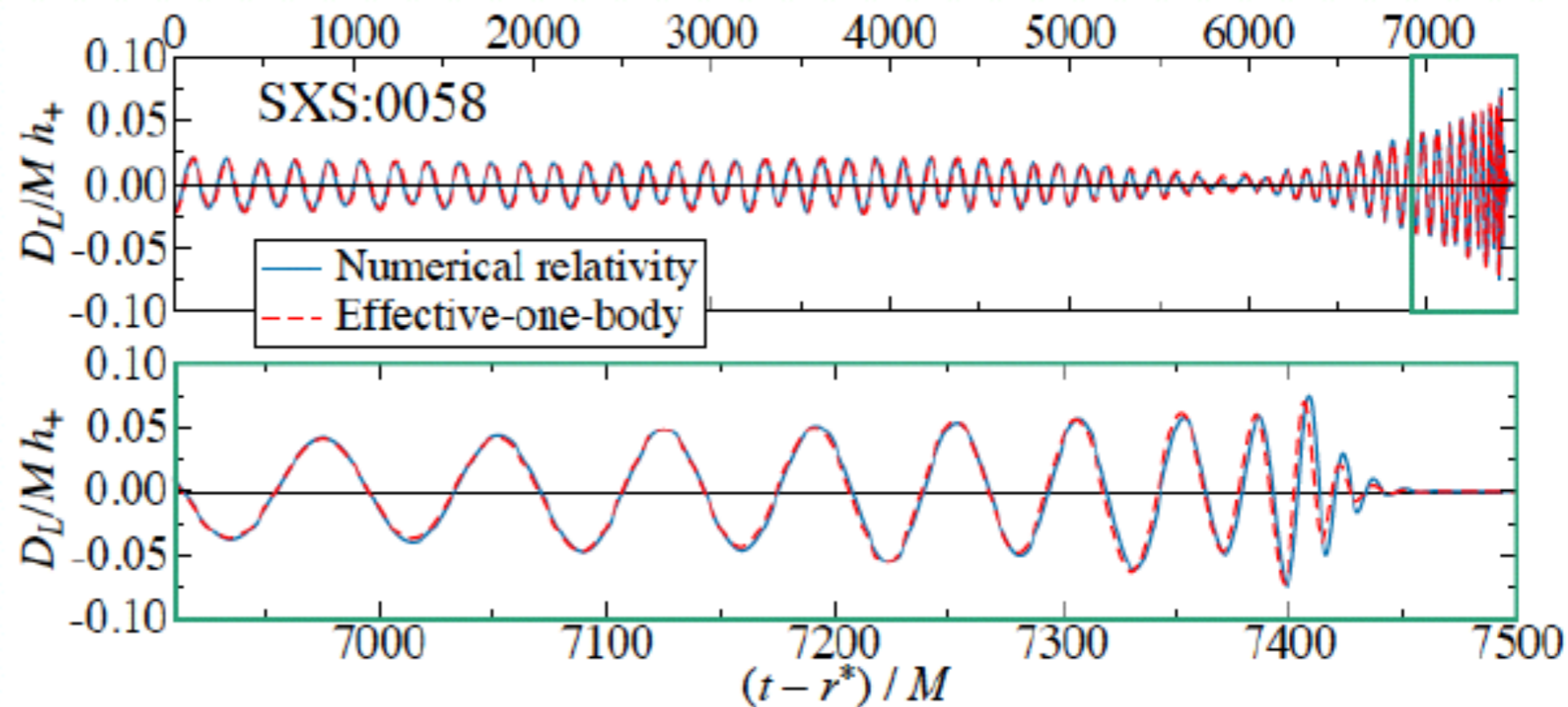
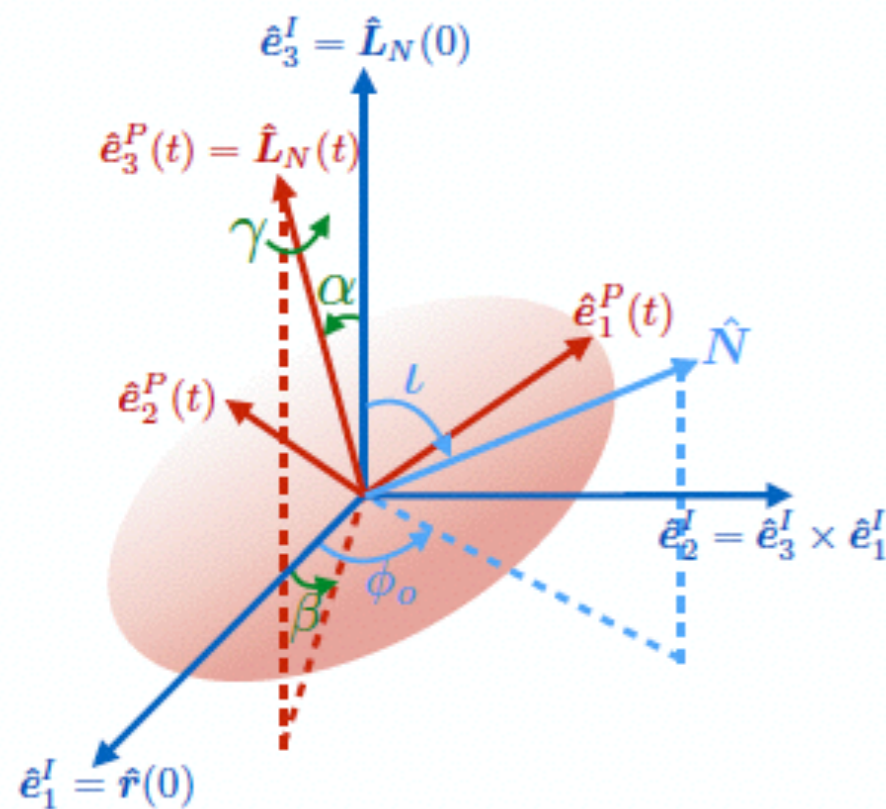
EOB waveform, spins are aligned with the orbital momentum



Taracchini et. al. 2013

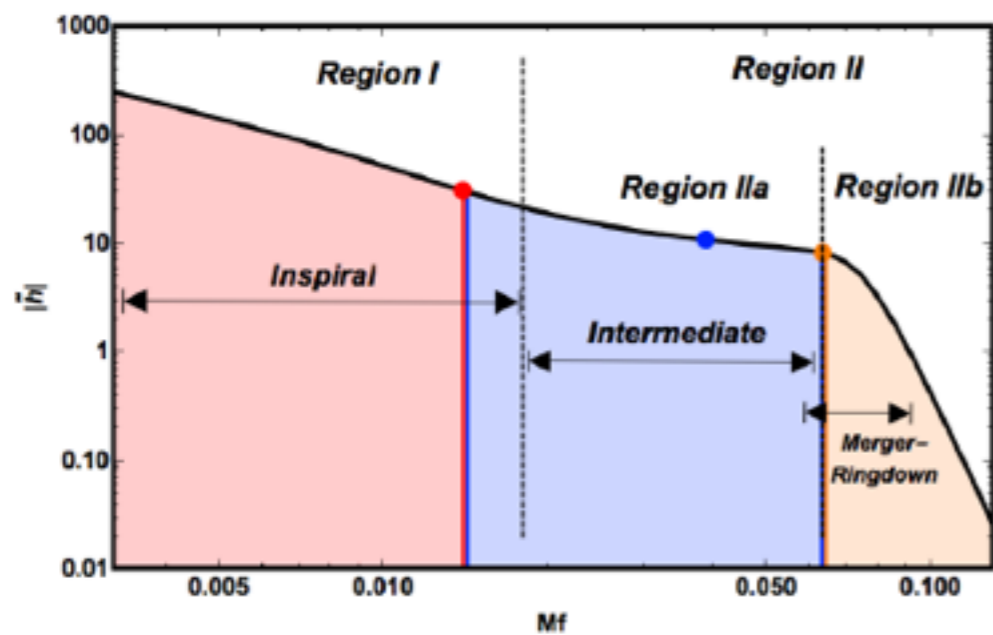
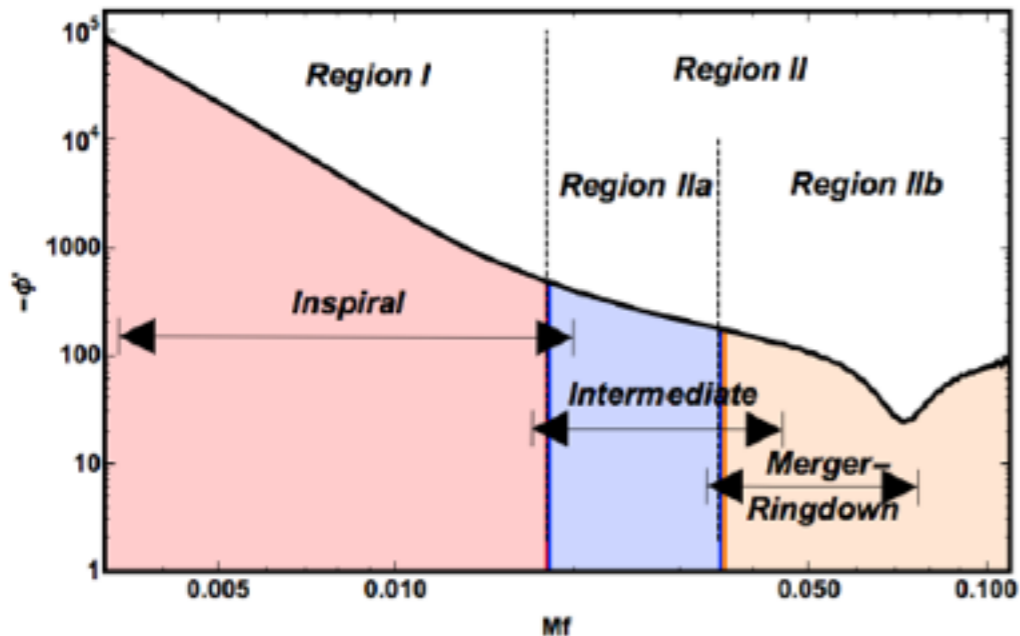
Precessing BH binary (EOB)

- Model precessing-frame waveforms with **calibrated IMR nonprecessing models**. **No recalibration** of inspiral of underlying nonprecessing models [Pan+13, Babak+(in prep)]

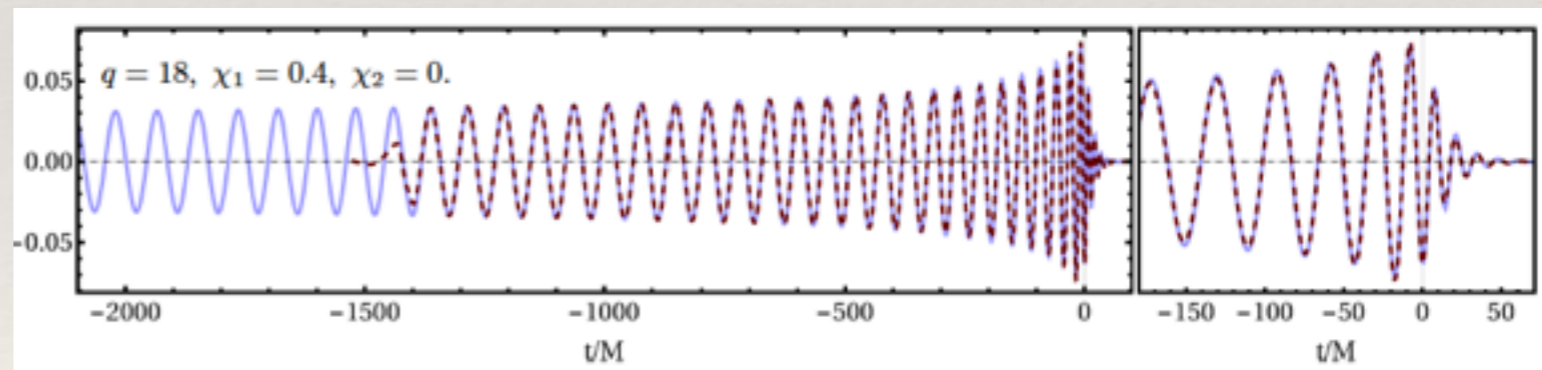


- 70 NR simulations** [SXS13] w/ mass ratios b/w 1 and 5, spins magnitudes up to 0.5, generic orientations [Babak+(in prep)]

IMRPhenomP



- Waveform constructed in the frequency domain
- Uses Post-Newtonian results for the early evolution (inspiral) of a binary (EOB)
- For merger-ringdown part: there is an analytical expression with free parameters which are calibrated to fit the NR data
- Precession is added by rotation taken from the Post-Newtonian evolution
- Very fast to generate



Khan et.al. 2015

Basic parameters of the BH binary

Distance: 440 Mpc ($z=0.09$)

$m_1 = 39$, $m_2 = 30$, remnant mass = 67 mass ratio ~ 0.8

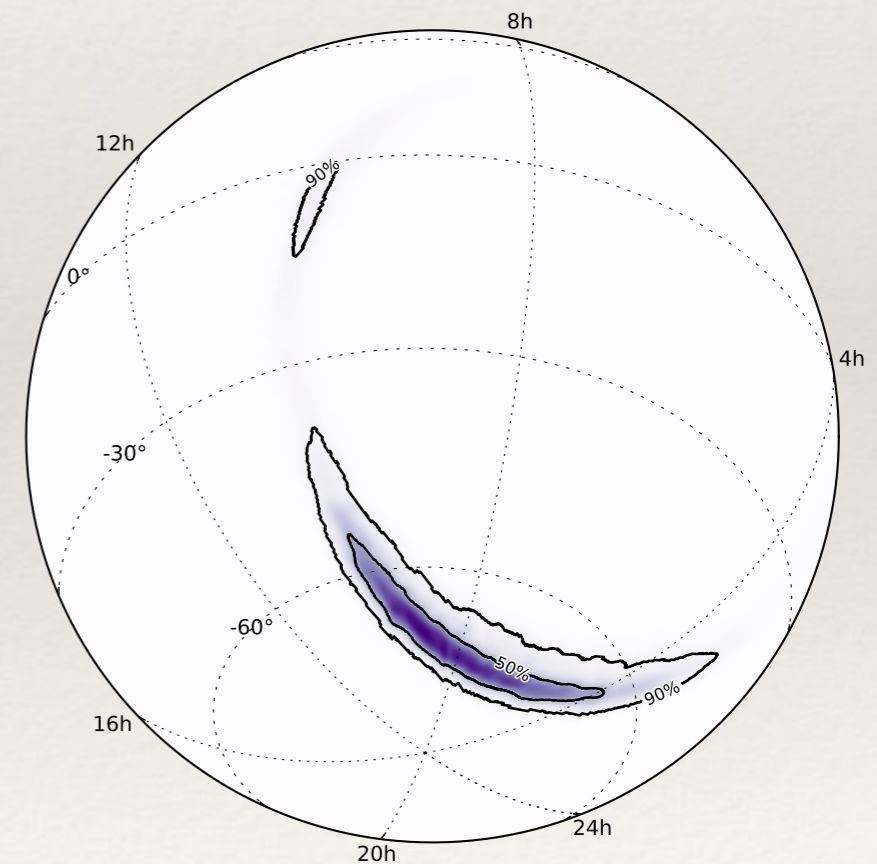
Position: face-off, south hemisphere, 600 sq.deg.

Duration (from 30Hz), ~ 200 ms, ~ 10 cycles Peak amplitude freq.: 150 Hz

QNM frequency: 250 Hz, damping time: 4 ms

Radiated energy: 2.25 M (between 30 and 240 Hz)

Peak luminosity: 3.6×10^{56} erg s $^{-1}$



Recovered parameters of the binary

	Non-precessing Model	Precessing Model
M/M_{\odot}	$70.3^{+5.3}_{-4.8}$	$70.7^{+3.9}_{-3.9}$
$M^{\text{source}}/M_{\odot}$	$65.0^{+5.0}_{-4.4}$	$64.6^{+4.1}_{-3.5}$
\mathcal{M}/M_{\odot}	$30.2^{+2.5}_{-1.9}$	$30.5^{+1.8}_{-1.7}$
$\mathcal{M}^{\text{source}}/M_{\odot}$	$27.9^{+2.3}_{-1.8}$	$27.9^{+1.8}_{-1.6}$
m_1/M_{\odot}	$39.4^{+5.5}_{-4.9}$	$38.3^{+5.5}_{-3.5}$
$m_1^{\text{source}}/M_{\odot}$	$36.3^{+5.3}_{-4.5}$	$35.1^{+5.2}_{-3.3}$
m_2/M_{\odot}	$30.9^{+4.8}_{-4.4}$	$32.2^{+3.6}_{-5.1}$
$m_2^{\text{source}}/M_{\odot}$	$28.6^{+4.4}_{-4.2}$	$29.5^{+3.3}_{-4.5}$
M_f/M_{\odot}	$67.1^{+4.6}_{-4.4}$	$67.4^{+3.4}_{-3.6}$
$M_f^{\text{source}}/M_{\odot}$	$62.0^{+4.4}_{-4.0}$	$61.6^{+3.7}_{-3.1}$
q	$0.79^{+0.18}_{-0.19}$	$0.84^{+0.14}_{-0.21}$
χ_{eff}	$-0.09^{+0.19}_{-0.17}$	$-0.03^{+0.15}_{-0.15}$
χ_p	—	$0.38^{+0.42}_{-0.28}$
a_1	$0.32^{+0.45}_{-0.28}$	$0.31^{+0.51}_{-0.28}$
a_2	$0.57^{+0.40}_{-0.51}$	$0.39^{+0.50}_{-0.35}$
a_f	$0.67^{+0.06}_{-0.08}$	$0.67^{+0.05}_{-0.05}$
D_L/Mpc	390^{+170}_{-180}	440^{+140}_{-180}
z	$0.083^{+0.033}_{-0.036}$	$0.093^{+0.028}_{-0.036}$
Δt	$6.94^{+0.50}_{-0.42}$	$6.94^{+0.48}_{-0.39}$

IMRPhenom

“Combined” spin along orbital angular momentum

$$\chi_{\text{eff}} = \left(\frac{\vec{S}_1}{m_1} + \frac{\vec{S}_2}{m_2} \right) \frac{\hat{\mathbf{L}}}{M}$$

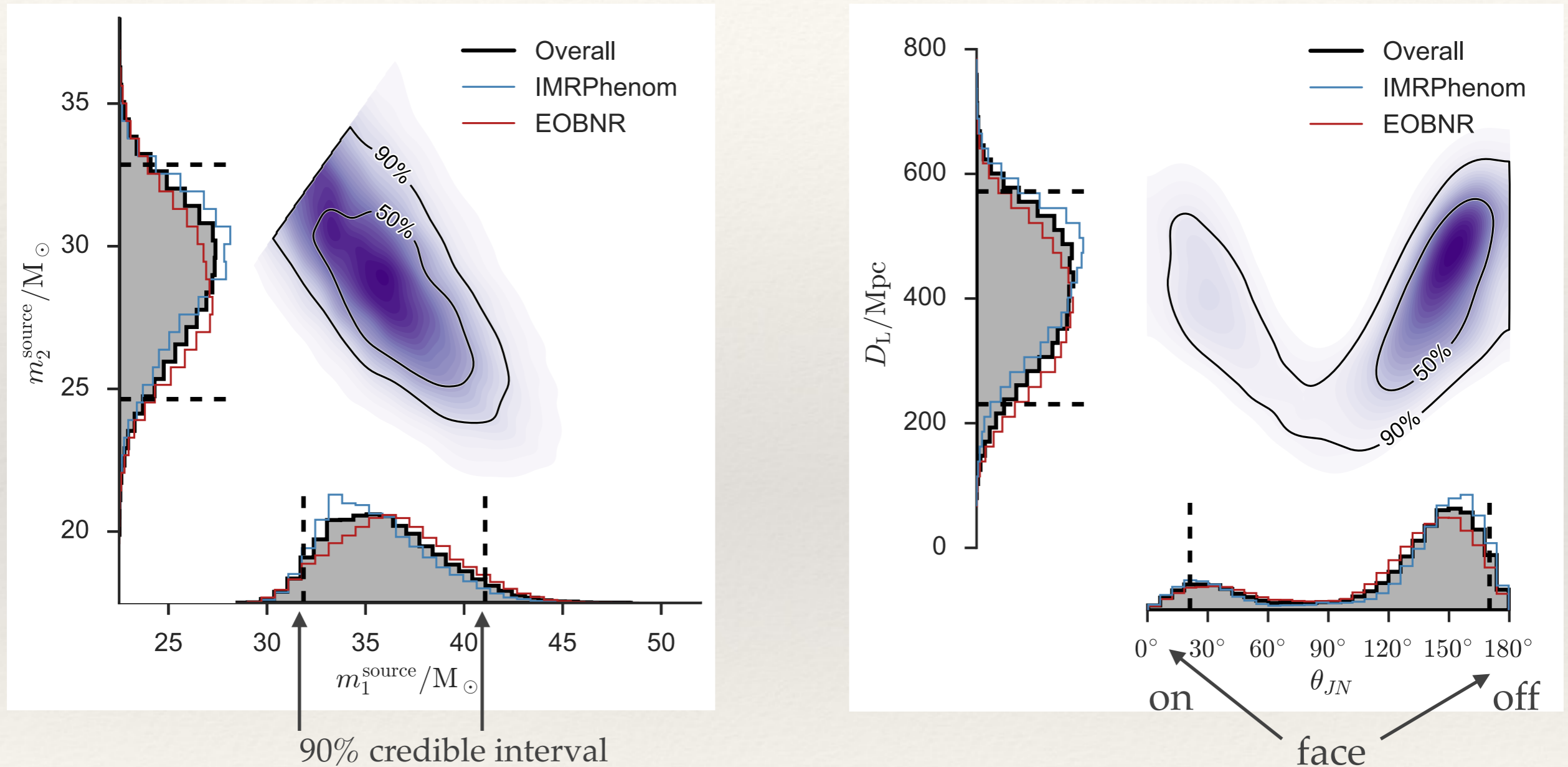
“Combined” spin components in the orbital plane

$$\chi_p = \frac{1}{B_1 m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp})$$

$$B_1 = 2 + \frac{3m_2}{2m_1}, \quad B_2 = 2 + \frac{3m_1}{2m_2}$$

Masses, distance, inclination

Posterior distribution function for masses, distance and orbital inclination: recovered in post-processing analysis using Bayesian techniques.

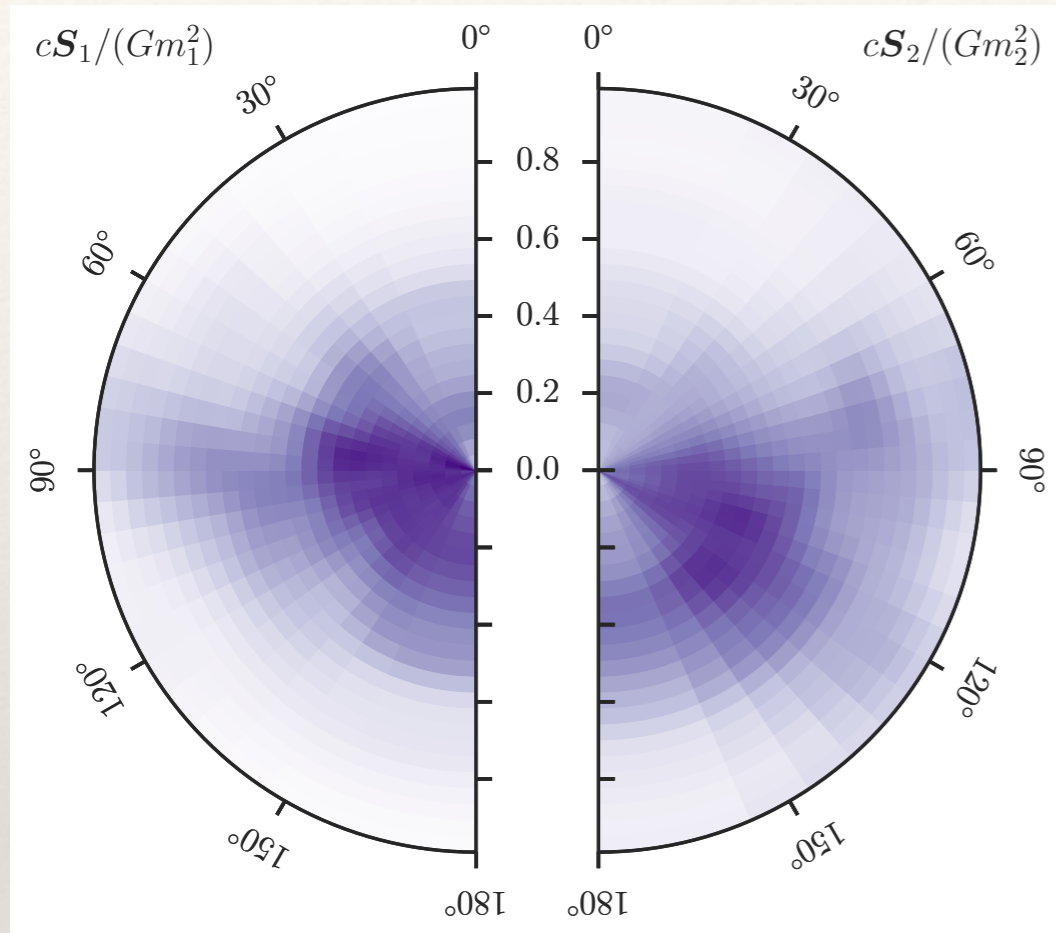


90% credible interval

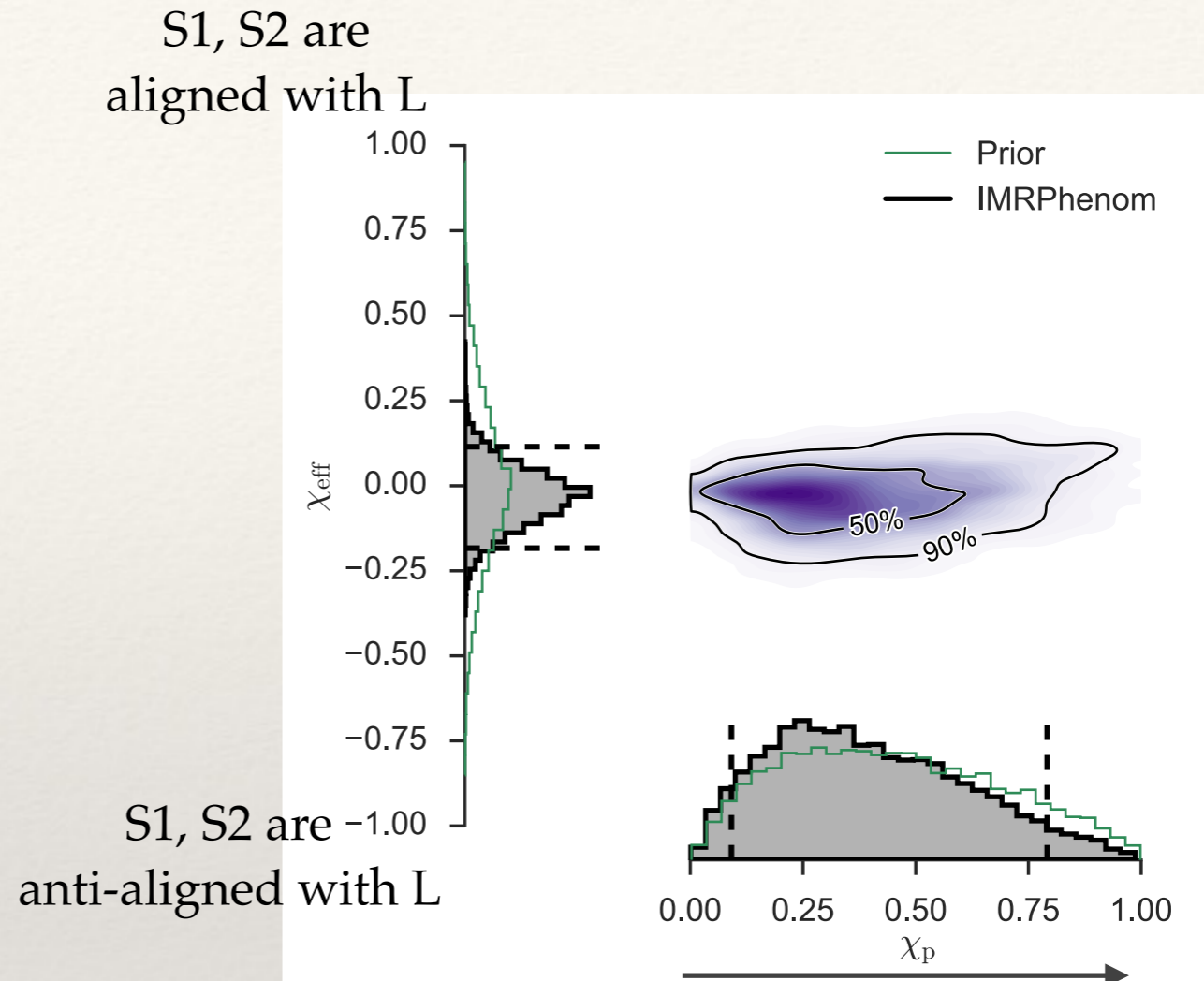
on face off

LVC arXiv:1602.03840

Spins (IMRPhenomP)



Slice orthogonal to the orbital plane



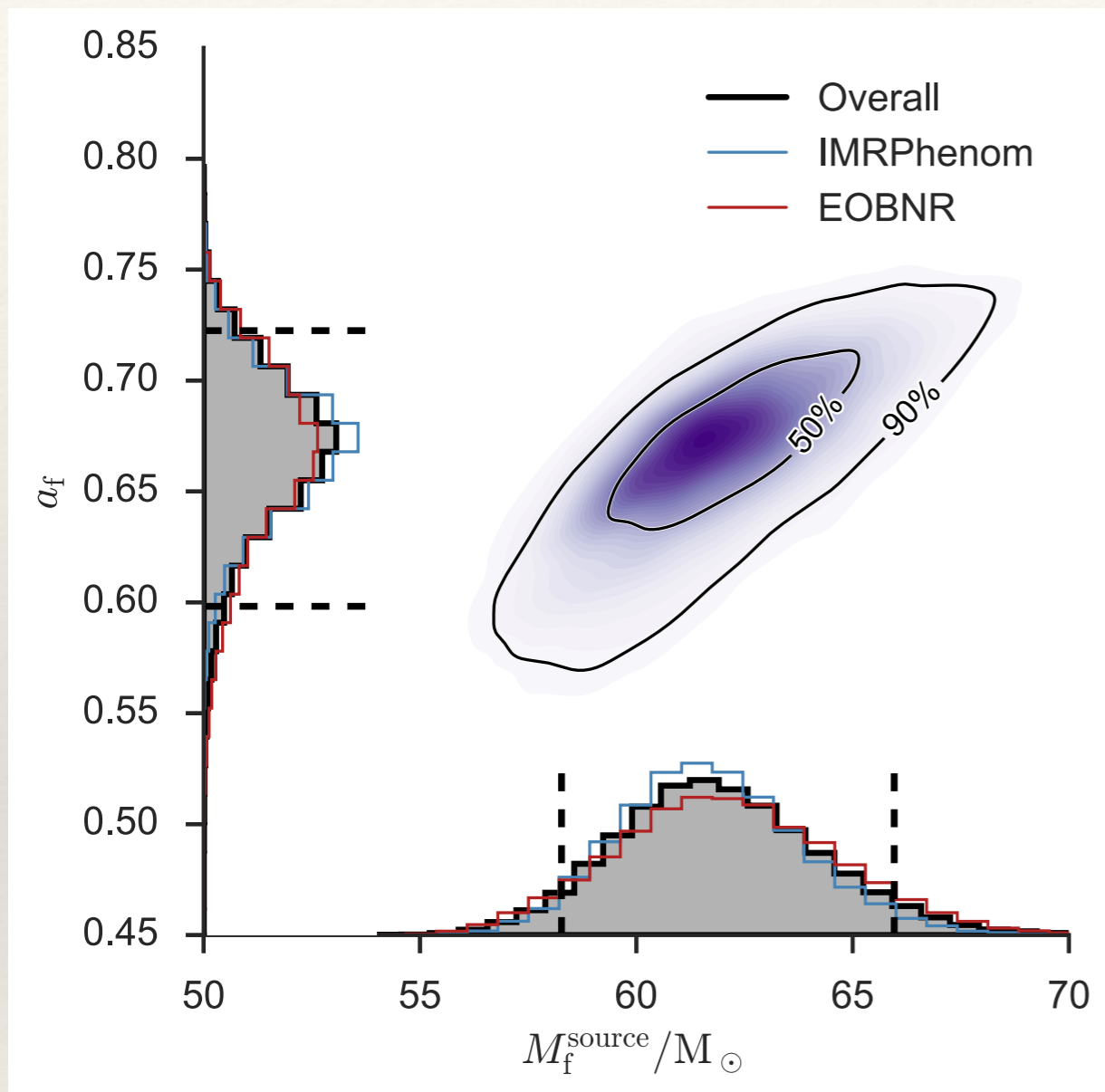
S1, S2 are aligned with L
S1, S2 are anti-aligned with L

No precession

Strong precession

Posterior distribution as reported by running data analysis with IMRPhenomP waveforms

Remnant BH

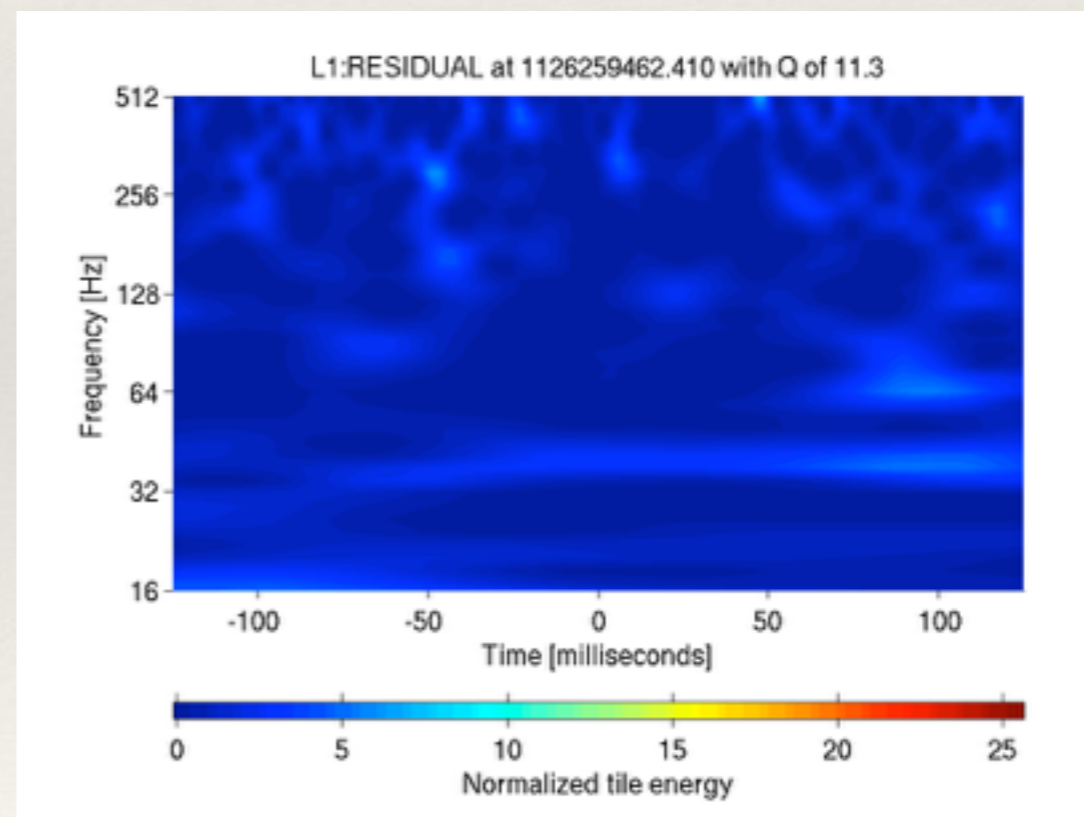
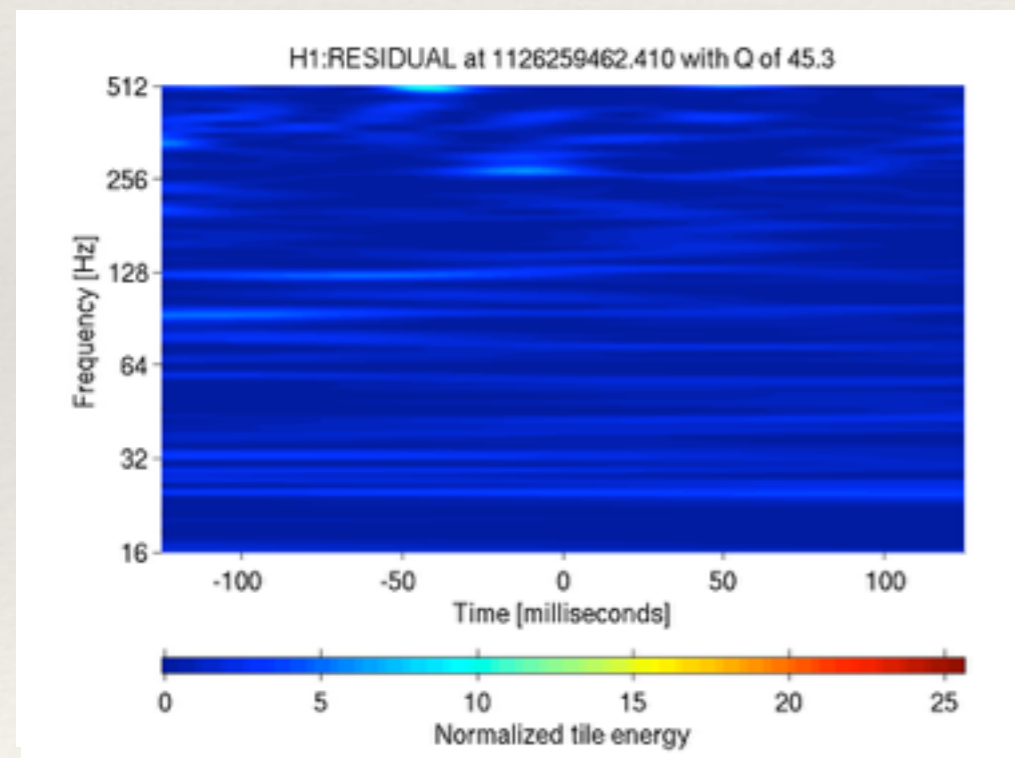
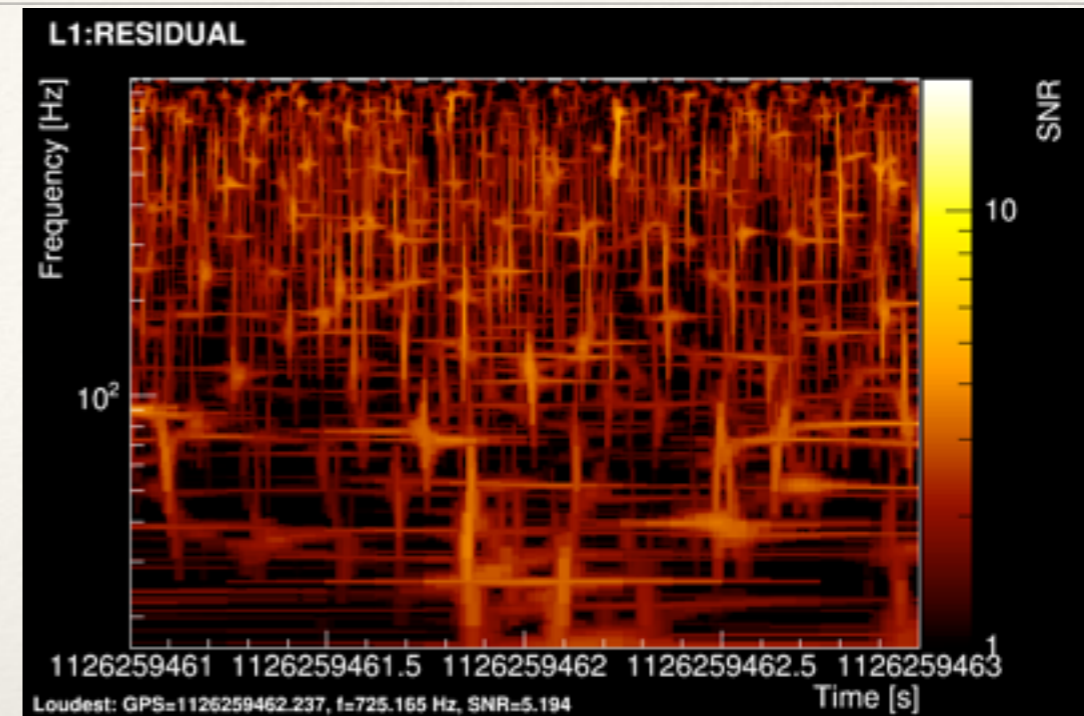
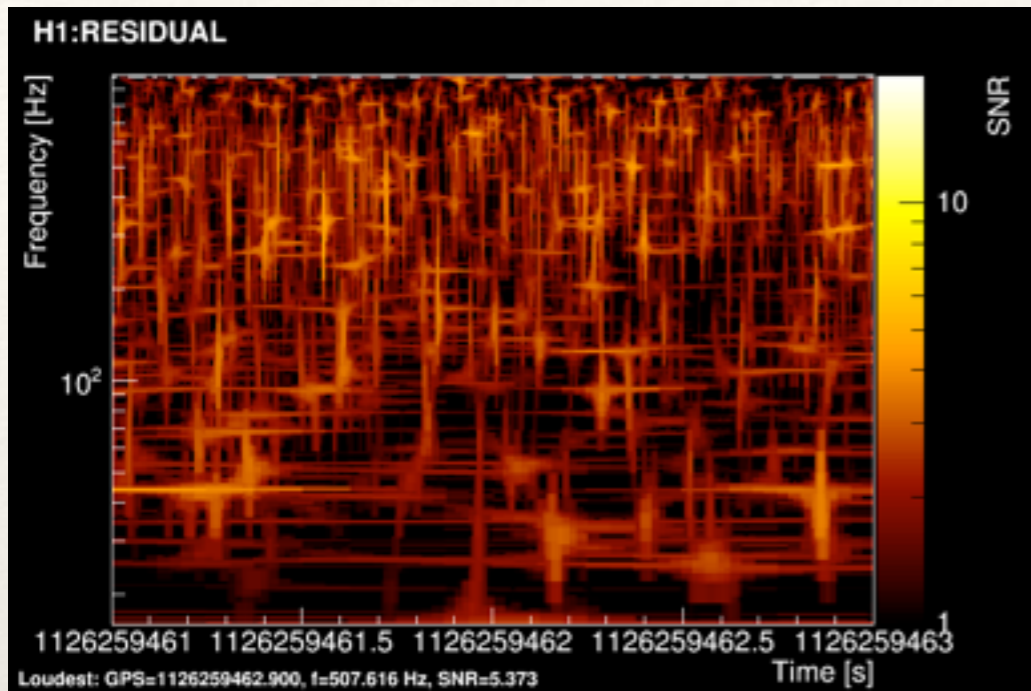


- Parameters of the remnant BH: final spin and mass
- Obtained using the fitting expression calibrated using NR data(Healy et.al. 2014)
- Mass deficit: Radiated energy: 2.25 M between 30 and 240 Hz

LVC arXiv:1602.03840
Healy et al 2014.

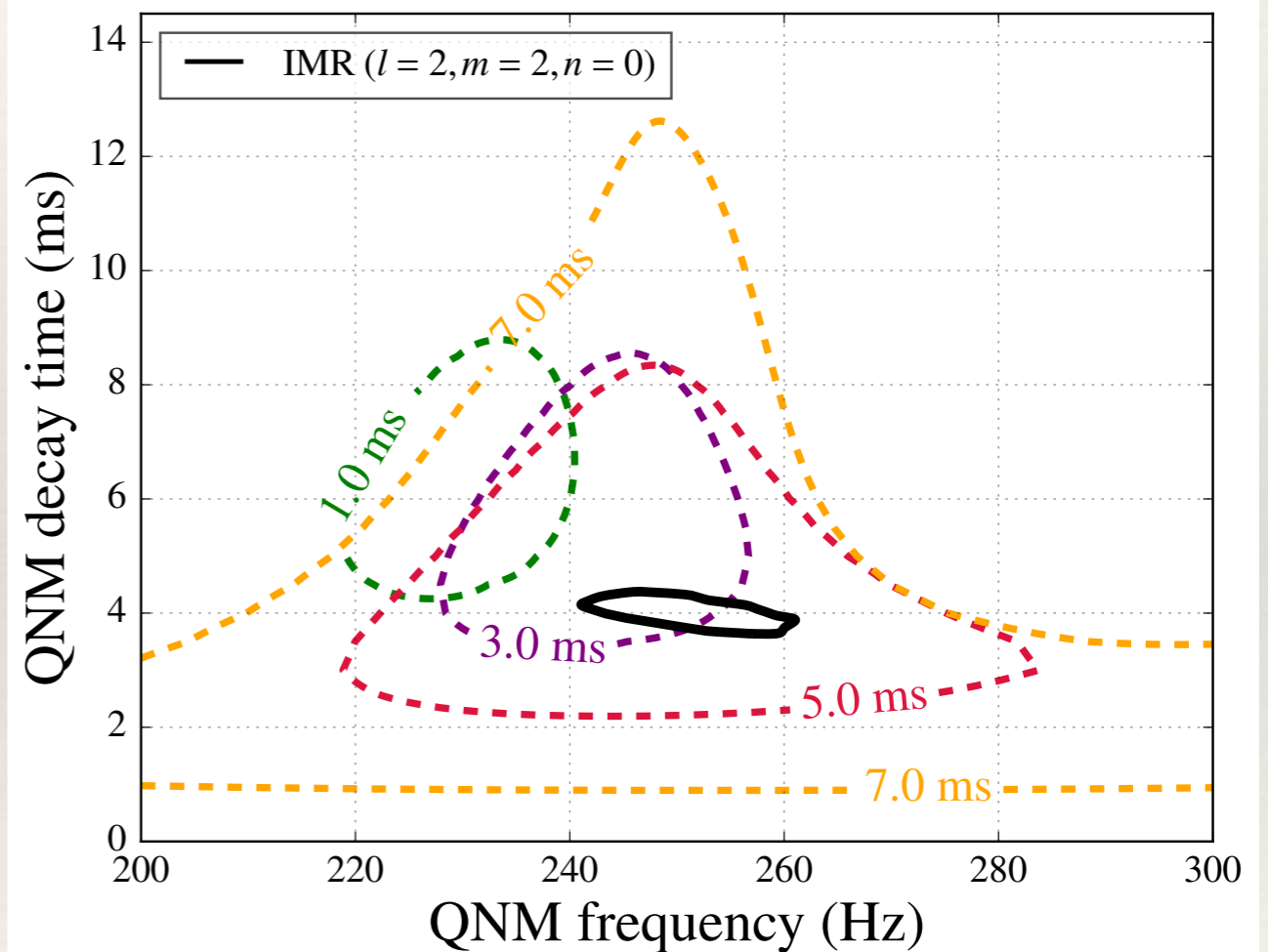
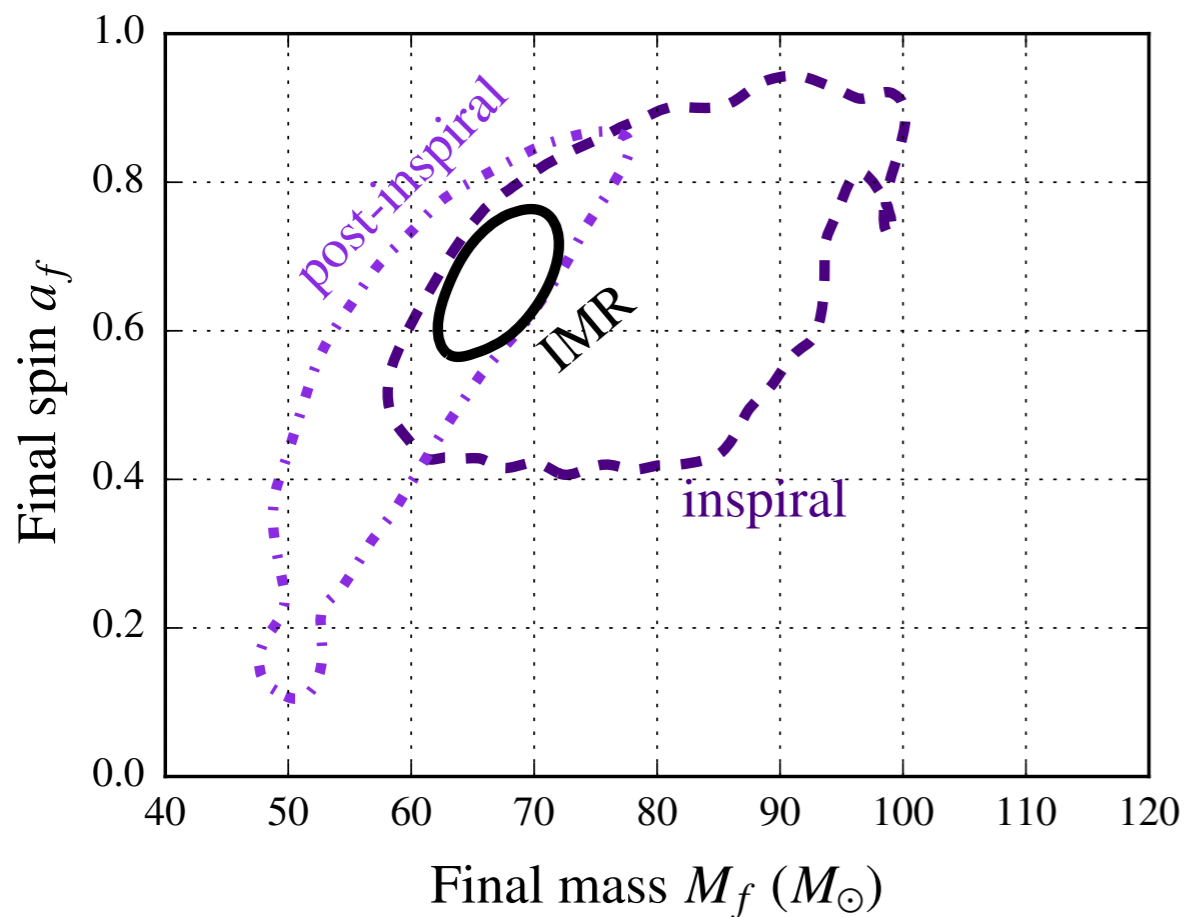


Consistency with GR (residuals study)

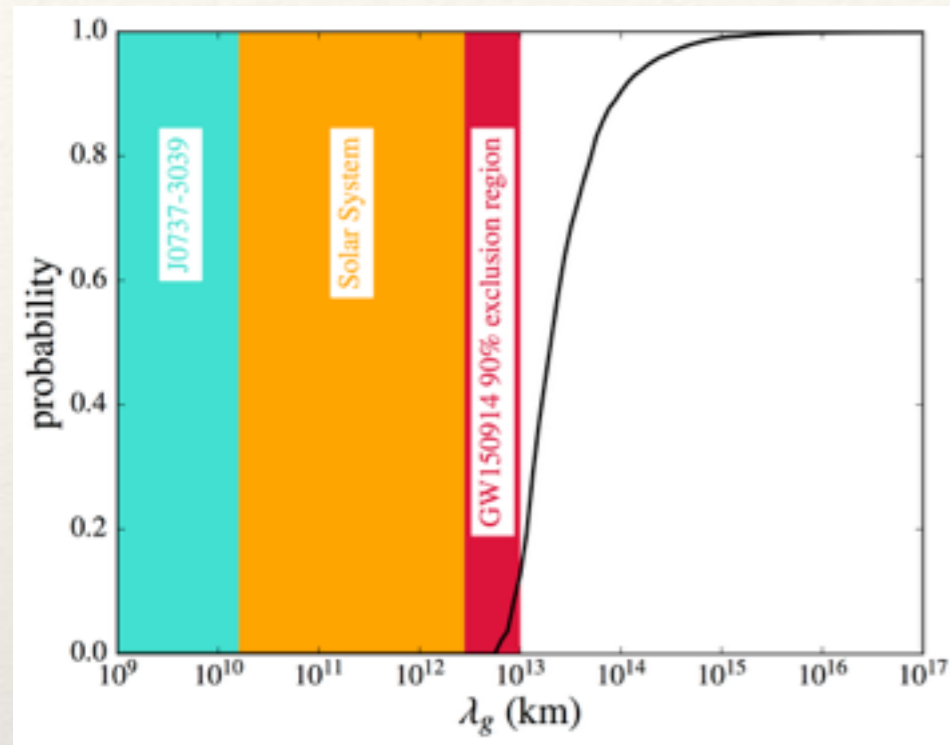


Consistency with GR predictions

- Study of consistency of inspiral (early orbital evolution) and merger parts of the signal: they show consistent estimation of the final mass and final spin of the remnant BH
- Quasi-normal modes produced during formation and relaxation of a remnant BH: superposition of the exponentially damped eigen modes of a BH. We attempt to identify the $n=0$ overtone (the longest lived mode) as a function of “post-merger” time



Constraining dispersion in the GW signal



$$\Delta t_a = (1 + z) \left[\Delta t_e + \frac{D}{2\lambda_g^2} \left(\frac{1}{f_e^2} - \frac{1}{f_e'^2} \right) \right]$$

$$\Delta t_e = t_e - t_e'$$

time of emission f_e
time of emission f_e'

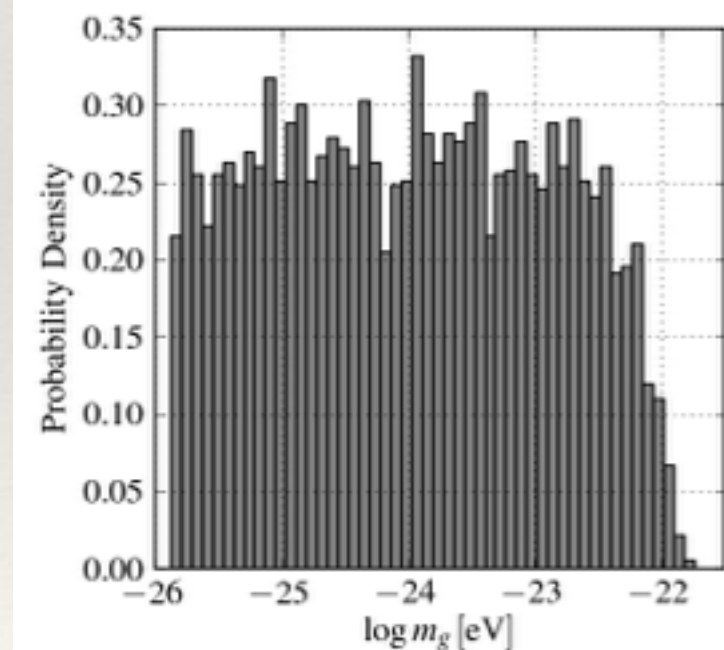
GR part

$$\tilde{h}(f) = A(f)e^{i\Psi(f)} \times e^{-i\beta_g(\pi\mathcal{M}_c f)^{-1}}$$

dispersion term

$$\beta \equiv \frac{\pi^2 D \mathcal{M}_c}{\lambda_g^2 (1+z)} \quad \lambda_g = h/(m_g c)$$

$$\lambda_g > 10^{13} \text{ km}; \quad m_g < 1.2 \times 10^{-22} \text{ eV}$$



Conclusion

- The gravitational wave event GW150914:
 - **First detection of gravitational wave signal**
 - **First detection of Black Hole binary system**
 - **First detection of the heaviest stellar-mass black hole**
- We have accurate waveforms (theoretical models) to reliably detect GW signals and estimate their parameters
- The observational bias (selection) prefers BH systems face-on/off, which in turn makes it hard to estimate well the spins and their orientation
- All consistency checks performed on the GW signal show **no indication of any deviation from General Relativity and binary Black Hole system**