COSMOLOGICAL DISTANCES

The “distance” to an object depends on the definition of distance. In ordinary physics we have different definitions which give equivalent results. In cosmology different definitions give different results.

METRIC OR PHYSICAL DISTANCE

- It is measured by rulers in the hand

$$\Delta r = \Delta r(\text{lagrangian coordinate})$$

$$l_{phys} = a(t_0)\Delta r \equiv r_p$$

COSMIC DISTANCE

- This distance is measured by light ray

$$l_c(t) = a(t)\int_{t_e}^{t} \frac{d\widehat{t}}{a(\widehat{t})}$$

Let us consider the motion of light rays in the expanding Universe. The light is moving along a straight line according to the law

$$ds = 0, \quad \text{where}$$

$$ds^2 = c^2 dt^2 - a^2(t)\left\{dr^2 + r^2 d\Omega^2\right\}$$

Let us assume that an observer is in the center of spherical coordinate system and the light rays move along radial coordinate. So, we can put

$$d\theta = 0, d\varphi = 0 \Rightarrow d\Omega = 0$$

So, we have three equations

$$d\theta = 0, d\varphi = 0, ds = 0$$

$$ds^2 = c^2 dt^2 - a^2(t)\left\{dr^2 + r^2 d\Omega^2\right\}$$

The last one is postulate of Special Relativity which is valid in the General Relativity too. One can resolve the metric equation

$$c^2 dt^2 - a^2(t) dr^2 = 0 \qquad \text{or}$$

$$dt = \pm a(t) dr$$

$$\int_{t_e}^{t_0} \frac{dt}{a(t)} = -r_0 + r_e$$

Here r_e is lagrangian distance to an extragalactic object

r_e is lagrangian distance to an extragalactic object, multiplying it by the *a(t)*

one will get the physical distance between the object and the observer. We will call this cosmic distance.

$$l_c = a(t_0)r_e = a(t_0)\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

THE ANGULAR SIZE DISTANCE

- Angular size distance is

$$l_a = a(t_e)r_e$$

The angular-size distance in ordinary physics determines as

$$l_a = \frac{l_{\text{object}}}{\Delta\varphi}$$

is ratio of an object size over the angular size of the object from the point of observation

Let suppose that at some distance from an observer is the object of size D. Coordinates of the ends of the object are (r_1, θ_1, φ_1) and (r_2, θ_2, φ_2). The light emitted by the dots at the ends of the object at time t_e reaches the observer at time t_0, after passing along the radial null geodesic. Therefore, two lines from two ends of the object intersect in the observer position with an angles $\Delta\theta$, $\Delta\varphi$. 2D metric at the moment of emission is

$$dl^2 = a^2(t_e) r_e^2 d\Omega^2$$

Therefore, physical size of the object is

$$l_{\text{object}} = a(t_e)r_e\Delta\varphi$$

and angular distance size of the object is

$$l_{\text{a}} = a(t_e)r_e = \frac{a(t_e)}{a(t_0)}a(t_0)\,r_e = \frac{l_c}{1+z}$$

THE BOLOMETRIC DISTANCE

- Bolometric distance is

$$l_b = \sqrt{\frac{L}{4\pi F}}$$

Here L is absolute luminosity, and F is the flux measured by an observer

Let $L(\nu_e, t_e)d\nu_e$ be the power emitted by an extragalactic object in the frequency band $d\nu_{e.}$ In a time dt_e the objects emits the numbers

$$\frac{L(\nu_e,t_e)d\nu_e dt_e}{h\nu_e}$$

Of photons inside the frequency interval ν_e and $\nu_\mathrm{e} + d\nu_e$. These photons spread out uniformly on a spherical front of increasing size without absorption. It means that the number of quanta is conserved. The spherical front has reached a coordinate radius r_0 from the object and it has an area

$$4\pi r_0^2 a^2(t_0).$$

The number of photons is the same, but energy of a photon decreases in *(1+z)* times.

So the energy per unit area received from a quasar is

$$\frac{L(\nu_e, t_e) d\nu_e dt_e}{4\pi r_0^2 a^2(t_0)(1+z)}$$

This energy is received in a time interval

$$dt_0 = (1+z) dt_e$$

Therefore

$$F(\nu_0, t_0)d\nu_0 = \frac{L(\nu_e, t_e)d\nu_e dt_e}{4\pi r_0^2 a^2(t_0)(1+z)^2}$$

Integration of both sides gives

$$F(t_0) = \frac{L(t_e)dt_e}{4\pi r_0^2 a^2(t_0)(1+z)^2}$$

or according to definition

$$l_b = \sqrt{\frac{L}{4\pi F}} = a(t_0)r_e(1+z)$$

The angular distance is easy to calculate from metric

$$l_a = a(t_e)r_e = \frac{l_c}{1+z}$$

and one can list the equations

$$l_c = a(t_0) r_e = a(t_0) \int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$l_a = \frac{l_c}{1+z}$$

$$l_b = l_c (1+z)$$

and one can derive the equation

$$l_a l_b = l_c^2$$

The cosmic distance is geometrical mean of the bolometric distance and the angular-size distance.

We shall find l_a, l_b, l_c as functions of known cosmological parameters (observable parameters). They are: red shift z, Hubble constant H_0, types of matter which fill our Universe. The main aim is to calculate l_c as function of these parameters, since two others are defined by multiplication and division of $(1+z)$.

$$l_c = -a(t_0)\int_{t_e}^{t_0} \frac{d\hat{t}}{a(\hat{t})}$$

$$l_c = \int_0^z (1+z)\,dt(z)$$

$$a(t) = \frac{a_0}{1+z} \Rightarrow \frac{da}{dt} = -\frac{a_0}{(1+z)^2}\frac{dz}{dt}$$

$$H(t)dt = -\frac{dz}{1+z}$$

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_{0m}(1+z)^3 + \Omega_q + \Omega_{0r}(1+z)^4\right]$$

$$r_c = -\int_0^z (1+z)\,dt(z) = \int_0^z \frac{dz}{H(z)}$$

$$r_c = \frac{c}{H_0}\int_0^z \frac{dz}{\sqrt{\Omega_{mo}(1+z)^3 + \Omega_q + \Omega_{ro}(1+z)^4}}$$