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SPHERICALLY SYMMETRIC COLLAPSE TO A POINT-LIKE STATE

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♢ A POINT MASS IN GENERAL RELATIVITY.

♠ Coalescence of binary black holes:

- G. Schäfer, Post-Newtonian Methods: Analytic Results on the Binary Problem in book: *Mass and Motion in General Relativity*, 167–210 (Springer, 2011);
- L. Blanchet, *LRR*, 17, 187 (2014);
- T. Damour, P. Jaranowski, G. Schäfer, *PLB*. 513, 147 (2001);
- and others.

- Extremely necessary for describing LIGO's and Virgo's discovery!
- At an *initial* step the black holes are modeled by *point-like* particles presented by Dirac's δ-function.
- Then consequent post-Newtonian approximations are used; excellent mathematics, regularization, etc
- The interpretation problems.
- A point-like description as a fundamental problem.
- A necessity of an exact presentation.

♠ AN EXACT PRESENTATION.

- The Schwarzschild BH as a point particle described by the Dirac δ -function!
- REQUIREMENTS:

(i) The true singularity has to be described by the world line $r = 0$ with the use of the Dirac $\delta(\mathbf{r})$-function.

(ii) The Schwarzschild solution has to be presented in the asymptotically flat form with appropriate (Newtonian) fall-off of potentials at spatial infinity.

(iii) To be consistent with a continuous spherically symmetric collapse trajectories of falling test particles have to achieve the true singularity continuously.

- The point (i) cannot be satisfied in the geometrical presentation of GR. The same physical reality can be described in various mathematical techniques. The field-theoretical methods in GR resolves the problems.
- OTHER REQUIREMENTS:

(iv) We require a so-called "η-causality" (property, when the physical light cone is inside the background light cone) at all the points of the background spacetime.

(v) We require a finite time for a free test particle in the background spacetime to achieve the true singularity.

♣ THE FIELD-THEORETICAL PRESENTATION OF GR.

Lagrangian of the gravity theory:

$$\mathcal{L} = \mathcal{L}(g^{\mu\nu},\, \phi^A) = \mathcal{L}^G(g^{\mu\nu}) + \mathcal{L}^M(g^{\mu\nu},\, \phi^A) \tag{1}$$

- ♠ ϕ^A – a set of tensor densities (matter fields);
- ♠ $\gamma^{\mu\nu}$ – Minkowski metric in curvelinear coordinates (background);
- ♠ $\bar{\mathcal{L}} = \mathcal{L}(\gamma^{\mu\nu})$ – Lagrangian of the background system.

Perturbations, $h^{\mu\nu}$ (the fields configuration - dynamic variables):

$$\sqrt{-g}g^{\mu\nu} = \sqrt{-\gamma}(\gamma^{\mu\nu} + h^{\mu\nu}); \tag{2}$$

Lagrangian for new, $\hat{h}^{\mu\nu} = \sqrt{-\gamma}h^{\mu\nu}$, φ^A, dynamic variables:

$$\mathcal{L}^{dyn} = \mathcal{L}\left(\gamma + h,\, \phi\right) - \hat{h}^{\mu\nu}\frac{\delta\bar{\mathcal{L}}}{\delta\hat{\gamma}^{\mu\nu}} - \bar{\mathcal{L}} \tag{3}$$

Variation with respect to $\hat{h}^{\mu\nu}$ leas to the field equations:

$$G^L_{\mu\nu} = \kappa(t^{\rm g}_{\mu\nu} + t^{\rm m}_{\mu\nu}) = \kappa t^{\rm tot}_{\mu\nu}\,, \tag{4}$$

The total energy-momentum tensor:

$$t^{\rm tot}_{\mu\nu} \equiv \frac{2}{\sqrt{\bar{g}}}\frac{\delta\mathcal{L}^{dyn}}{\delta\gamma^{\mu\nu}}\,, \qquad \bar{\nabla}_\nu t^{\mu\nu}_{\rm tot} = 0\,. \tag{5}$$

♢ The works in the field-theoretical formulation in GR:

- S.Deser, GRG, 1, 9 (1970);
- L.P. Grishchuk, A.N. Petrov and A.D. Popova, Commun. Math. Phys., 94, 379 (1984);
- L.P. Grishchuk and A.N. Petrov, ZhETF, 92, 9 (1987);
- A.D. Popova and A.N. Petrov, IJMPA, 3, 2651 (1988);
- A.N. Petrov, S.M. Kopeikin, R.R. Lompay and B. Tekin, "Metric Theories of Gravity: Perturbations and Conservation Laws" (Germany: De Gruyter, 2017).

♣ GAUGE TRANSFORMATIONS AND GAUGE INVARIANCE

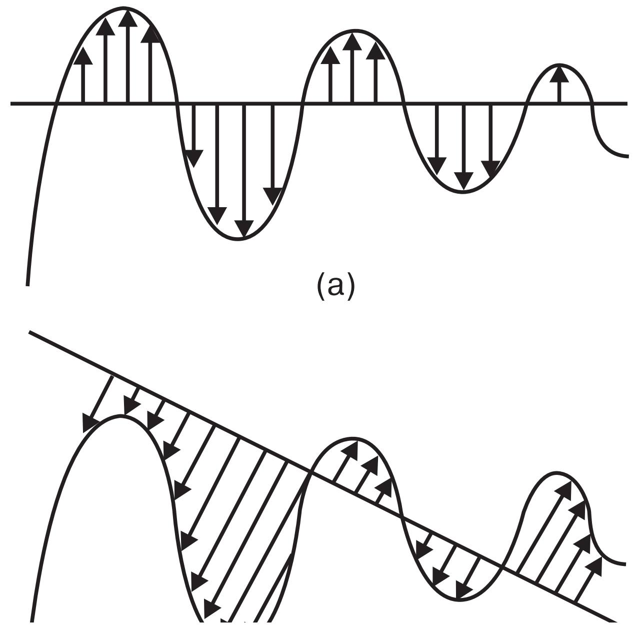
The same solution to the Einstein equations can be written in another coordinate chart, say, $\{x'^{\alpha}\}$. The corresponding decomposition is

$$\sqrt{-g'}g'^{\mu\nu}(x') \equiv \sqrt{-\gamma'}\left(\gamma'^{\mu\nu}(x') + h'^{\mu\nu}(x')\right). \tag{6}$$

Then, after the shifting in the frame $\{x'^{\alpha}\}$ from points with values of the coordinates x'^{α} to points with values x^{α} and after equalizing $\gamma'^{\mu\nu}(x) = \gamma^{\mu\nu}(x)$, one gets

$$\sqrt{-g'}g'^{\mu\nu}(x) \equiv \sqrt{-\gamma}\left(\gamma^{\mu\nu}(x) + h'^{\mu\nu}(x)\right). \tag{7}$$

The interpretation is as follows. They are related to the same solution to the Einstein equations; for both of these decompositions the same background presented by the metric $\gamma_{\mu\nu}$ is chosen by different ways. One concludes that the fields $h^{\mu\nu}$ and $h'^{\mu\nu}$ describe the same physical reality, only they and connected by gauge transformations.



◊ Perturbations connected by gauge transformations.

♢ Gauge transformations (symbolic description):

Full (finite) gauge transformations for the dynamical variables:

$$h'^{\mu\nu} = h^{\mu\nu} + \sum_{k=1}^{\infty} \frac{1}{k!} \pounds_\xi^k \left(\gamma^{\mu\nu} + h^{\mu\nu}\right), \qquad \phi'^A = \phi^A + \sum_{k=1}^{\infty} \frac{1}{k!} \pounds_\xi^k \phi^A. \tag{8}$$

Gauge transformations in linear gravity theory on a flat background (Lorenzian coordonates):

$$h^{\mu\nu} = g^{\mu\nu} - \eta^{\mu\nu}, \implies h'^{\mu\nu} = h^{\mu\nu} - \pounds_\xi \eta^{\mu\nu}, \implies h'^{\mu\nu} = h^{\mu\nu} + \partial^\mu \xi^\nu + \partial^\nu \xi^\mu \tag{9}$$

♢ Invariance with respect to gauge transformations:

- **Lagrangain is gauge-invariant up to a divergence on the background equations.**
- **THE FIELD-THEORETICAL EQUATIONS ARE GAUGE-INVARIANT ON THE BACKGROUND EQUATIONS AND ON THEMSELVES.**
- **The energy-momentum tensor is NOT gauge-invariant:**

$$\kappa t'^{\rm tot}_{\mu\nu} = \kappa t^{\rm tot}_{\mu\nu} + G^L_{\mu\nu}(\delta h)$$

♢ A point particle in the Newtonian gravity;

- $\varphi = m/r$ – the Newtonian potential for a point mass:
- The Newtonian gravity equation:

$$\Delta\varphi = -4\pi\rho(\vec{r}) \implies \tag{10}$$

- $\rho(\vec{r}) = m\delta(\vec{r})$ – the the mass density for a point mass.

♢ The Schwarzschild solution as a field configuration in Minkowski space.

- The Schwarzschild solution:

$$ds^2 = (1 - r_g/r)\, c^2dt^2 - (1 - r_g/r)^{-1}\, dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{11}$$

- The Einstein equations:

$$G_{\mu\nu} = \kappa T_{\mu\nu} \implies T_{\mu\nu} = ??? \quad \text{(not satisfactory)}. \tag{12}$$

- The field-theoretical form of the GR equations,

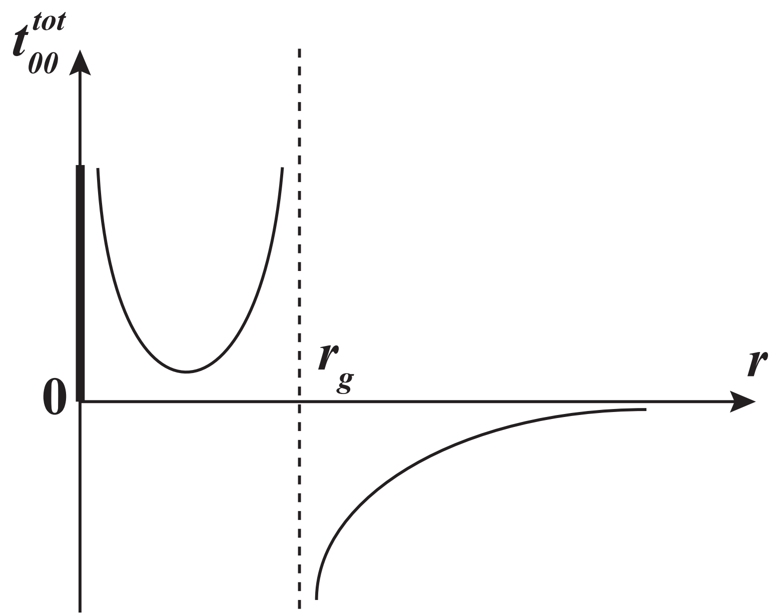
$$G^L_{\mu\nu} = t^{tot}_{\mu\nu}\,. \tag{13}$$

- The background Minkowski space:

$$d\bar{s}^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{14}$$

- The field configuration:

$$h^{00}_s = -\frac{r_g/r}{1 - r_g/r}\,, \qquad h^{11}_s = \frac{r_g}{r}\,. \tag{15}$$



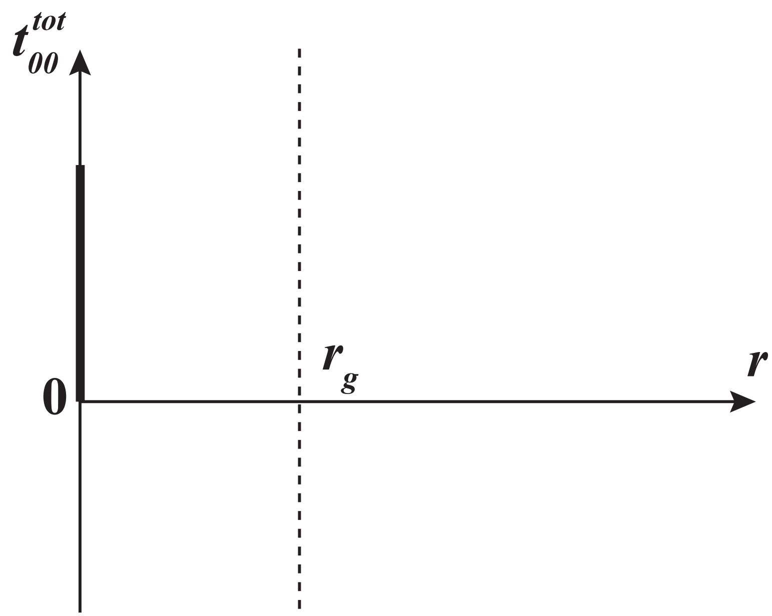
The energy density of the gravitational field and sources.

- The break in $h_s^{\mu\nu}$ and t_{00}^{tot} corresponds to a break in geodesics:
  THE REQUIREMENT (iii) IS NOT HOLD!
- The coordinate transformation, like $cdt \to cdt + f(r)dr$ applied to physical metric $g_{\mu\nu}$ and a consequent choose of the same background as Minkowski space

$$d\overline{s}^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{16}$$

  changes the field configuration — it is interpreted as the gauge transformation.

♠ It is necessary to find a more appropriate gauge fixing.



♠ The Eddington-Finkelstein gauge fixing:
ALL THE REQUIREMENT ARE SATISFIED!

- The Schwarzschild solution:

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2c\,\frac{r_g}{r}\, dr dt - \left(1 + \frac{r_g}{r}\right) dr^2 - r^2 d^2\Omega. \tag{17}$$

- in Minkowski space: $d\bar{s}^2 = c^2 dt^2 - dr^2 - r^2 d^2\Omega$.

- The field configuration:

$$h_e^{00} = \frac{r_g}{r}\,, \qquad h_e^{01} = -\frac{r_g}{r}\,, \qquad h_e^{11} = \frac{r_g}{r}\,. \tag{18}$$

- Energy-momentum:

$$t_{00}^{tot} = mc^2\delta(\mathbf{r})\,, \qquad t_{11}^{tot} = -mc^2\delta(\mathbf{r})\,, \qquad t_{AB}^{tot} = -\tfrac{1}{2}\bar{g}_{AB}\, mc^2\delta(\mathbf{r})\,. \tag{19}$$

♢ A generalization Of the Eddington-Finkelstein gauge fixing for the Schwarzschild solution

♠ Coordinates transformations applied to the Eddington-Finkelstein frame:

$$cdt \rightarrow cdt + f(r_g/r)dr. \tag{20}$$

♠ Construction of field configurations with the background:

$$d\bar{s}^2 = c^2 dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta . d\phi^2 \right) \tag{21}$$

♠ Required properties of the related field configurations:

- **the true singularity is placed at $r = 0$ by the δ-function ;**
- **the field variables (perturbations) are asymptotically flat;**
- **regularity at the horizon.**

♠ A general gauge fixing

• **The Schwarzschild solution:**

$$ds^2 = \left(1-\frac{r_g}{r}\right)c^2dt^2 - 2\left[\frac{r_g}{r} + \left(1-\frac{r_g}{r}\right)f\right]cdtdr \\ - \left[\left(1+\frac{r_g}{r}\right) - 2\frac{r_g}{r}f - \left(1-\frac{r_g}{r}\right)f^2\right]dr^2 - r^2d\Omega^2\,. \tag{22}$$

• **The field configuration:**

$$\begin{aligned} h_f^{00} &= \frac{r_g}{r} - 2\frac{r_g}{r}f - \left(1-\frac{r_g}{r}\right)f^2\,, \\ h_f^{01} &= -\frac{r_g}{r} - \left(1-\frac{r_g}{r}\right)f\,, \\ h_f^{11} &= \frac{r_g}{r}\,. \end{aligned} \tag{23}$$

• **The energy-momentum components:**

$$\begin{aligned} t_{00}^{tot} &= mc^2\delta(\mathbf{r}) - 4\pi r_g\delta(\mathbf{r})\left[2\left(f+\frac{r_g}{r}f'\right) + 2ff' - f^2 - 2\frac{r_g}{r}ff'\right] \\ &\quad + \left[4f'^2 + \left(f''-f'^2\right)\frac{r_g}{r} - 4ff' + ff''\left(1-\frac{r_g}{r}\right)\right]\frac{r_g^2}{r^4}\,, \end{aligned} \tag{24}$$

$$t_{11}^{tot} = -mc^2\delta(\mathbf{r})\,, \tag{25}$$

$$t_{AB}^{tot} = -\tfrac{1}{2}\gamma_{AB}\, mc^2\delta(\mathbf{r}); \qquad A,\ldots = 2,3\,. \tag{26}$$

♠ RESTRICTIONS FOR $f = f(r_g/r)$:

- **The requirement (i) is fulfilled - the true singularity is modeled by δ-function.**
- **The requirement (ii) of the Newtonian asymptotic behaviour:**

$$f(r_g/r)|_{r\to\infty} \sim (r_g/r)^{\alpha}; \qquad \alpha > 1/2. \tag{27}$$

- **The requirement (iii) of the continuous geodesics at $0 < r \leq \infty$:**

$$|f| < N; \qquad \text{smooth and monotonic for arbitrary large positive } N \tag{28}$$

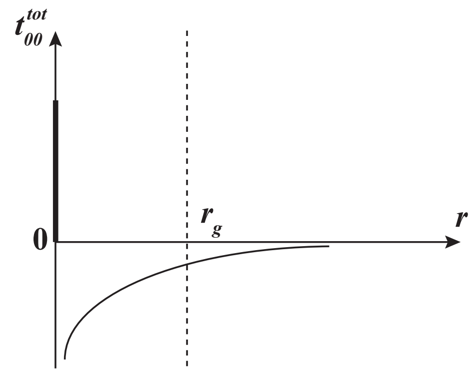
♠ Additional restrictions for $f = f(r_g/r)$:

- **The η-causality requirement (iv):**

$$|f(r_g/r)|_{r\to\infty} < \frac{2r_g}{r}; \qquad |f|_{r_g<r<\infty} \leq \frac{2r_g/r}{1 - r_g/r}. \tag{29}$$

- **The requirement (v) of a finite time of achieving the true singularity:**

$$|f|_{r\to 0} < N. \tag{30}$$



♠ A particular gauge fixing $f = -\frac{r_g}{r}$

- **The Schwarzschild solution:**

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2\,\frac{r_g^2}{r^2}\, c\, dt\, dr - \left(1 + \frac{r_g}{r}\right)\left(1 + \frac{r_g^2}{r^2}\right) dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\ .$$

- **The field configuration:**

$$h_f^{00} = \frac{r_g}{r} + \frac{r_g^2}{r^2} + \frac{r_g^3}{r^3}\,, \qquad h_f^{01} = -\frac{r_g^2}{r^2}\,, \qquad h_f^{11} = \frac{r_g}{r}\,; \tag{31}$$

- **The energy-momentum components:**

$$\begin{aligned} t_{00}^{tot} &= mc^2\delta(\mathbf{r}) + mc^2\,\frac{r_g}{r}\left(1 + \frac{3}{2}\frac{r_g}{r}\right)\delta(\mathbf{r}) - \frac{mc^2}{4\pi}\,\frac{r_g}{r^4}\left(1 + 3\frac{r_g}{r}\right)\ , \\ t_{11}^{tot} &= -mc^2\delta(\mathbf{r})\,, \\ t_{AB}^{tot} &= -\tfrac{1}{2}\gamma_{AB}\, mc^2\delta(\mathbf{r})\,; \qquad A,\ B = 2,\ 3. \end{aligned} \tag{32}$$

♢ CONTINUOUS COLLAPSE OF A DUST CLOUD

- J.R. Oppenheimer and H. Snyder, Phys. Rev., 56, 455 (1939) -
- The intrinsic and extrinsic solutions has to matched by the noncontradictive way - it is a problem:
- Y. Kanai, M. Siino and A. Hosoya, Prog. Theor. Phys., 125, 1053 (2011).

- The extrinsic Painlevé-Gullstrand coordinates:

$$ds^2 = \left(1 - \frac{2m}{r}\right) c^2 dt^2 - 2\sqrt{\frac{2m}{r}} dr\, c dt - dr^2 - r^2 d\Omega^2 \,. \tag{33}$$

- The generalized intrinsic Painlevé-Gullstrand coordinates:

$$ds^2 = \left(1 - \frac{4}{9}\frac{r^2}{(ct)^2}\right) c^2 dt^2 + \frac{4}{3}\frac{r}{ct} dr\, c dt - dr^2 - r^2 d\Omega^2 \,. \tag{34}$$

- Both of the solutions are matched smoothly automatically!

♠ Appilication of the field-theoretical tools is not sensible because the requirement of the point (ii) is not hold.

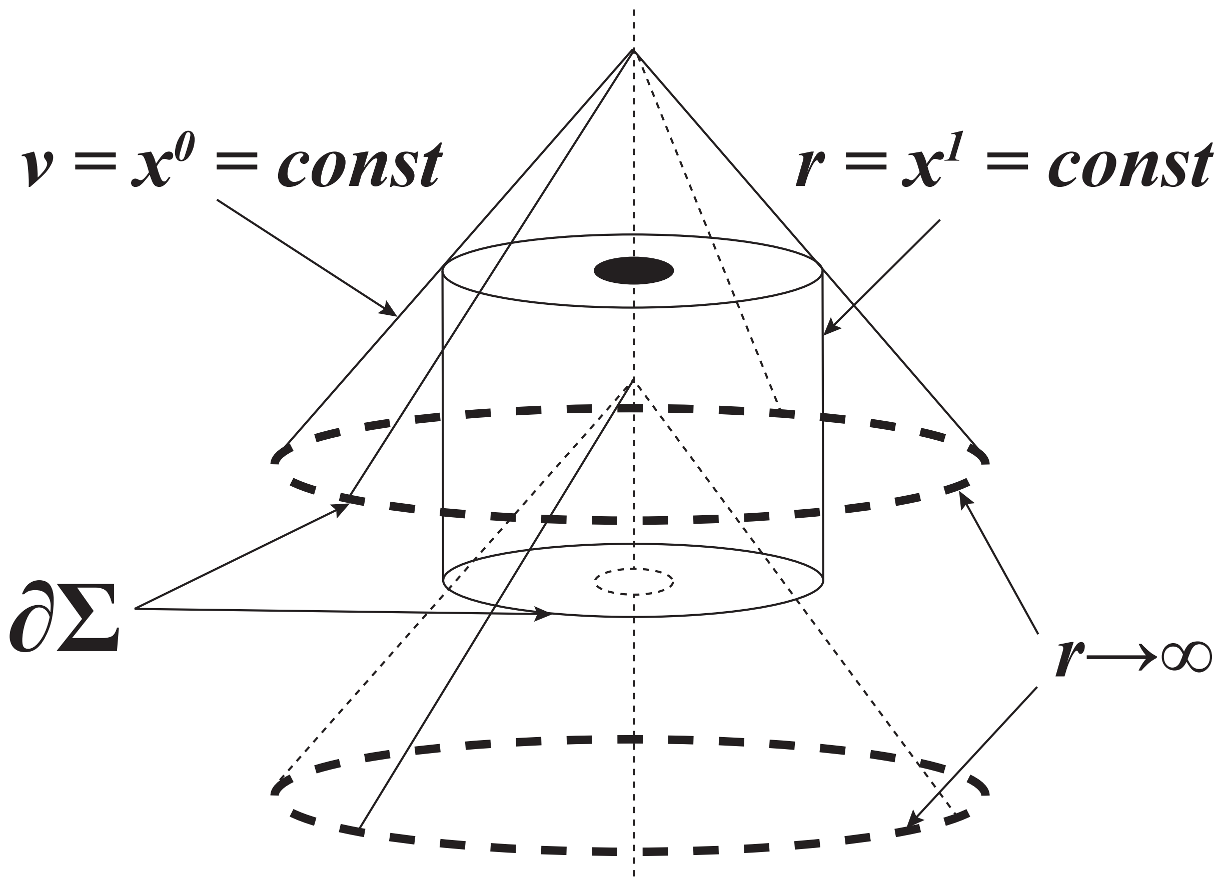


Рис. 1: Collapse of the dust cloud to a point.

♢ An appropriate change of the PG gauge fixing for a collapsing matter solution

♠ Coordinates transformations from the PG-like frame to the EF-like frame:

$$cdt \rightarrow cdt + \frac{(r_g/r)^{1/2}}{1+(r_g/r)^{1/2}}dr, \tag{35}$$

♠ Coordinates transformations from the PG-like frame to a general frame:

$$cdt \rightarrow cdt + \left(\frac{(r_g/r)^{1/2}}{1+(r_g/r)^{1/2}} - f(r_g/r)\right)dr = cdt + F(r_g/r)dr. \tag{36}$$

♠ Construction of field configurations with the background:

$$d\bar{s}^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta . d\phi^2\right) \tag{37}$$

♠ Required properties of the related field configurations:

- requirements $(i)-(iii)$, (v) are satisfied with the above requirements for f;
- the requirement (iv) are satisfied with the additional permissible restrictions for F in the intrinsic region

♢ Announce of the monograph:

Petrov, Alexander N. / Kopeikin, Sergei M. / Lompay, Robert R. / Tekin, Bayram

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