

# **СИЛЬНЫЕ МАГНИТНЫЕ ПОЛЯ В ФИЗИКЕ НЕЙТРИНО, КОСМОЛОГИИ И АСТРОФИЗИКЕ**

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# План доклада

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- Осцилляции нейтрино во внешних полях (в т.ч. в магнитном поле)
  - Магнитные и гипермагнитные поля в ранней вселенной
  - Астрофизические магнитные поля
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# **Глава 1**

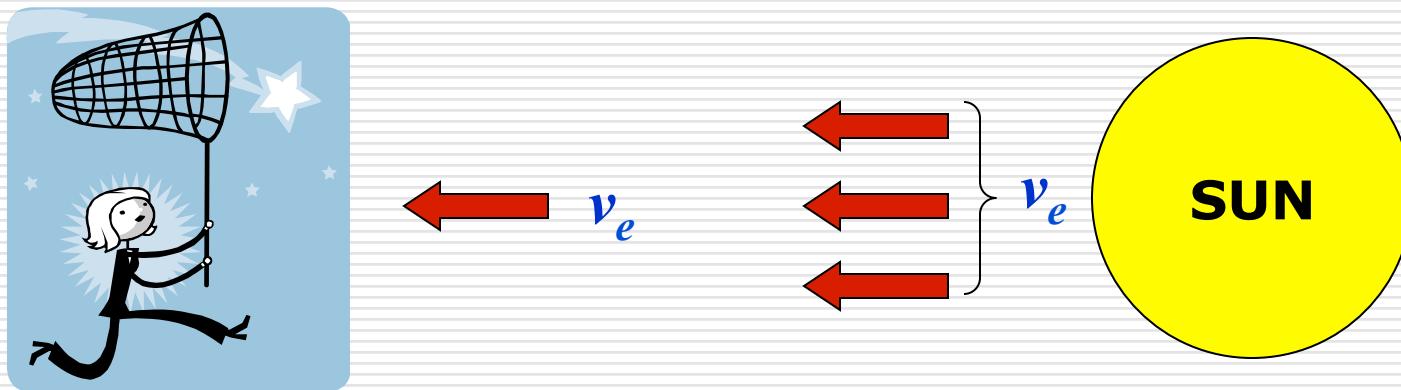
# **ОСЦИЛЛЯЦИИ НЕЙТРИНО В ВЕЩЕСТВЕ ПОД ДЕЙСТВИЕМ СИЛЬНОГО МАГНИТНОГО ПОЛЯ**

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# Experimental evidence of neutrino oscillations

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- We know how many electron neutrinos should be emitted by the Sun
- One observes less particles than predicted
- Therefore some of solar neutrinos disappear or are converted into particles invisible for us



# Mechanisms of neutrino oscillations

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- *Neutrino flavor oscillations.* Electron neutrinos are converted into muon or  $\tau$ -neutrinos
- *Neutrino spin and spin-flavor oscillations.* Left-handed electron neutrinos are converted into right-handed electron (or other flavor) neutrinos

$$\nu_e \rightleftharpoons \nu_{\mu,\tau}$$

$$\nu_e^L \rightleftharpoons \nu_e^R$$

$$\nu_e^L \rightleftharpoons \nu_{\mu,\tau}^R$$

# Flavor and mass eigenstates

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- There are flavor neutrinos which participate in weak interactions. These particles do not have definite masses.
- There are mass eigenstates which are the superposition of flavor neutrinos. These particles have definite masses.

$$\{\nu_\lambda\} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \vdots \end{pmatrix}$$
$$\{\psi_a\} = \begin{pmatrix} \psi_1 \text{ with mass } m_1 \\ \psi_2 \text{ with mass } m_2 \\ \vdots \end{pmatrix}$$

# Evolution of mass eigenstates

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- Since mass eigenstates have definite masses, the evolution of  $\psi_a$  is particularly simple

$$\psi_a(t) = e^{-iE_a t} \psi_a(0),$$

$$E_a = \sqrt{\mathbf{p}^2 + m_a^2}$$

# Evolution of flavor neutrinos

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- Flavor neutrinos are the superposition of massive neutrinos

$$\begin{aligned} \nu_\lambda(t) &= \sum_a U_{\lambda a} \psi_a(t) \\ &= \sum_{a\lambda'} \exp(-iE_a t) U_{\lambda a} U_{a\lambda'}^\dagger \nu_{\lambda'}(0), \\ U_{\lambda a} &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \end{aligned}$$

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# Transition and survival probabilities

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- Supposing that neutrinos are relativistic particles  $t \approx x$  (and  $|\mathbf{p}| \approx E$ ), we get transition and survival probabilities to find  $\nu_\mu$  or  $\nu_e$  in the initial  $\nu_e$  beam

$$P_{\nu_e \rightarrow \nu_\mu}(x) = \left| \langle \nu_\mu | \nu_e(x) \rangle \right|^2 = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2}{4E} x \right),$$

$$P_{\nu_e \rightarrow \nu_e}(x) = 1 - P_{\nu_e \rightarrow \nu_\mu}(x)$$

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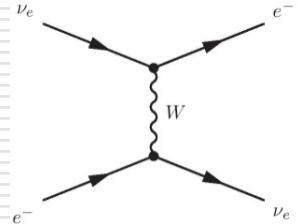
# Difficulties of the standard approach

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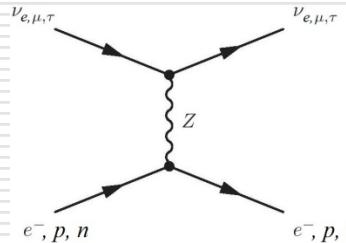
- Do different neutrino eigenstates have equal energies of equal momenta or equal velocities?
  - Do oscillations happen in time or in space?
  - Is it necessary to treat neutrinos as spinor particles?
  - Should one take into account the coordinate dependence of the neutrino wave function?
  - How can one describe oscillations of non-relativistic neutrinos?
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# Interaction of neutrinos with background matter

Charged currents interaction



$$\mathcal{L}_{eff}^{(CC)} = -\frac{G_F}{\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1-\gamma^5) \nu_e] \cdot [\bar{e} \gamma_\mu (1-\gamma^5) e]$$



Neutral currents interaction

$$\mathcal{L}_{eff}^{(NC)} = -\frac{G_F}{\sqrt{2}} [\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5) \nu_\alpha] \cdot [\bar{f} \gamma_\mu (I_{3L}^{(f)} (1-\gamma^5) - 2Q^{(f)} \sin^2 \theta_W) f]$$

After the averaging over the background fermions, we get:

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha \gamma^\mu (1-\gamma^5) \nu_\alpha \cdot f_\mu, \quad f_\mu = \sum_{f=e,p,n} [q_f^{(1)} j_\mu^{(f)} + q_f^{(2)} \lambda_\mu^{(f)}],$$

$$j^\mu = n_0 u^\mu, \quad \lambda^\mu = n_0 \left[ (\vec{\zeta} \mathbf{u}), \vec{\zeta} + \frac{\mathbf{u}(\vec{\zeta} \mathbf{u})}{1+u^0} \right], \quad u^\mu = (u^0, \mathbf{u})$$

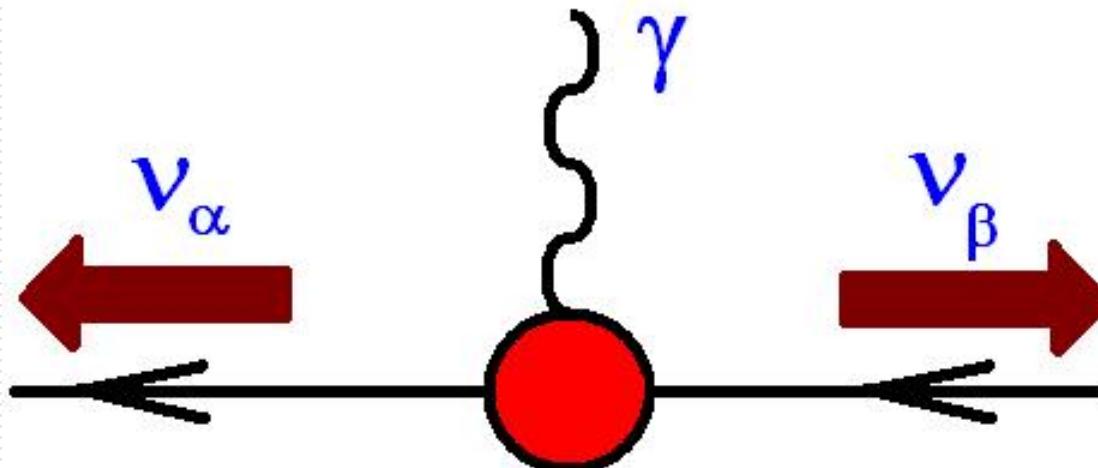
For Majorana neutrinos  
we should replace

$$\begin{aligned} \bar{\nu}_\alpha \gamma^\mu (1-\gamma^5) \nu_\alpha &\rightarrow \\ &\rightarrow -2 \bar{\nu}_\alpha \gamma^\mu \gamma^5 \nu_\alpha \end{aligned}$$

# Neutrino electromagnetic properties

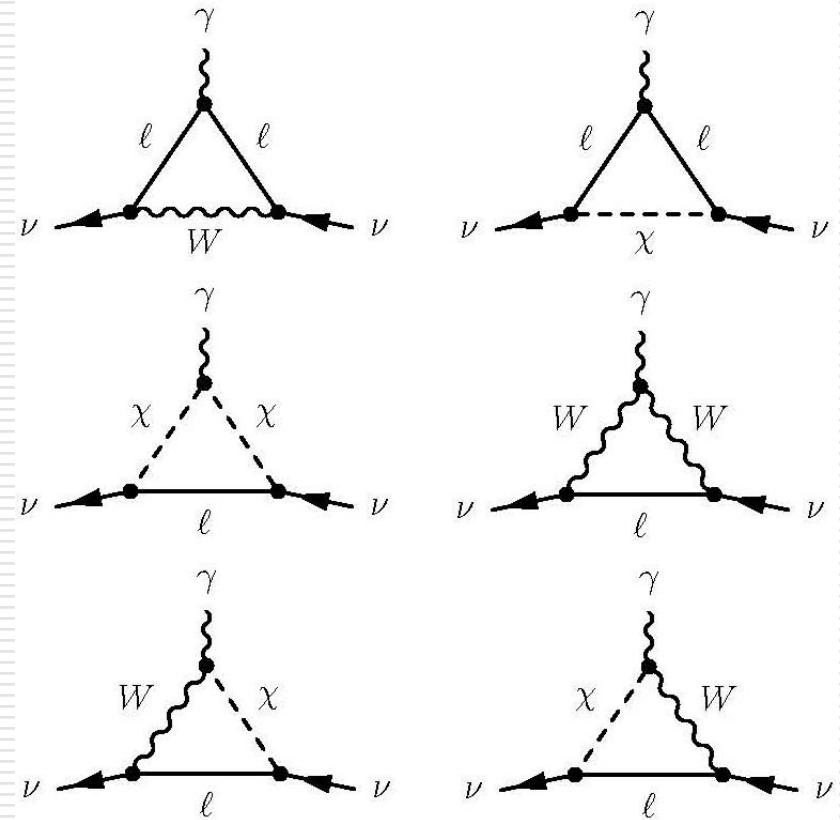
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- Despite neutrinos are electrically neutral particles they can have magnetic moments
- Non-relativistic limit  $(1/2) \sigma_{\mu\nu} F^{\mu\nu} \rightarrow \sigma \mathbf{B}$
- Both neutrino flavor,  $\nu_\beta \rightarrow \nu_\alpha$ , and helicity,  $\nu_L \rightarrow \nu_R$ , change in an external electromagnetic field



# Neutrino magnetic moment

- Minimally extended standard model supplied with SU(2) singlet right handed neutrino
- Magnetic moment is small  
 $\sim 10^{-19} \mu_B (m_\nu / \text{eV})$



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# **EVOLUTION OF FIRST QUANTIZED FLAVOR NEUTRINOS**

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# Initial condition problem

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- Flavor neutrinos Lagrangian

$$\mathcal{L} = \sum_{\lambda=\alpha\beta} \bar{\nu}_\lambda i\gamma^\mu \partial_\mu \nu_\lambda - \sum_{\lambda,\lambda'=\alpha\beta} m_{\lambda\lambda'} \bar{\nu}_\lambda \nu_{\lambda'}$$

$$- \sum_{\lambda=\alpha\beta} \bar{\nu}_\lambda \gamma^L_\mu \nu_\lambda f_\lambda^\mu - \frac{1}{2} \sum_{\lambda,\lambda'=\alpha\beta} M_{\lambda\lambda'} \bar{\nu}_\lambda \sigma_{\mu\nu} \nu_{\lambda'} F^{\mu\nu}$$

- Initial conditions
- Fields distributions at subsequent moments of time

$$\nu_\lambda(\mathbf{r}, t=0) = \xi_\lambda(\mathbf{r})$$
$$\nu_\lambda(\mathbf{r}, t) = ? \text{ at } t > 0$$

# Результаты

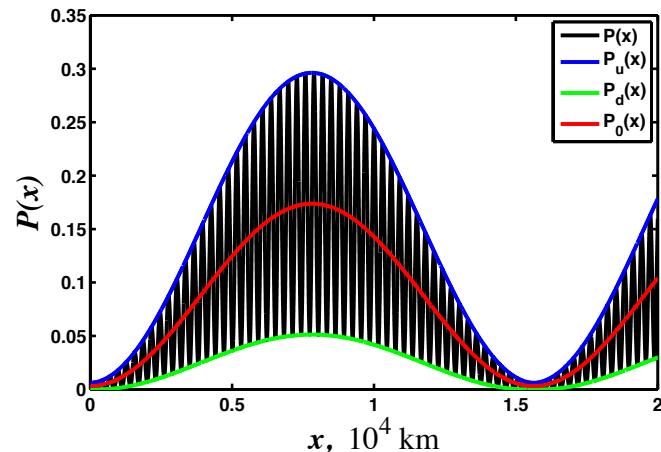
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- Начальная задача для системы массивных флейворных нейтрино, которые могут быть как дираковскими так и майорановскими, была решена для случая распространения нейтрино в вакууме, в веществе и в магнитном поле.
  - Стандартное квантовомеханическое описание осцилляций нейтрино было воспроизведено в рамках подхода основанного на теории поля.
  - Поправки к квантовомеханическому описанию осцилляций были получены.
  - На основе общих формул рассмотрены некоторые астрофизические приложения.
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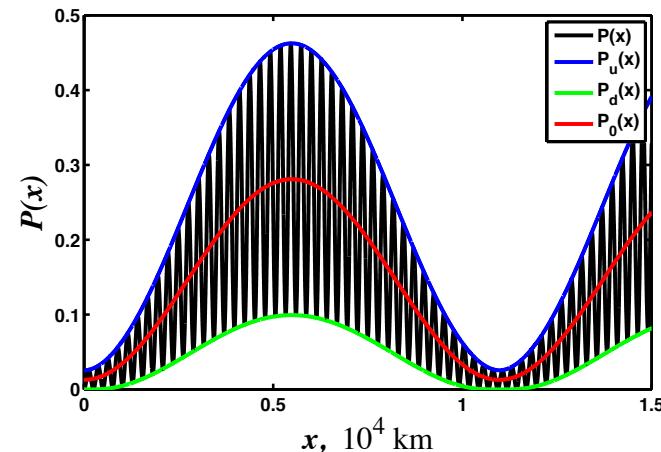
# Спин-флайворные осцилляции нейтрино в расширяющейся оболочке после взрыва сверхновой

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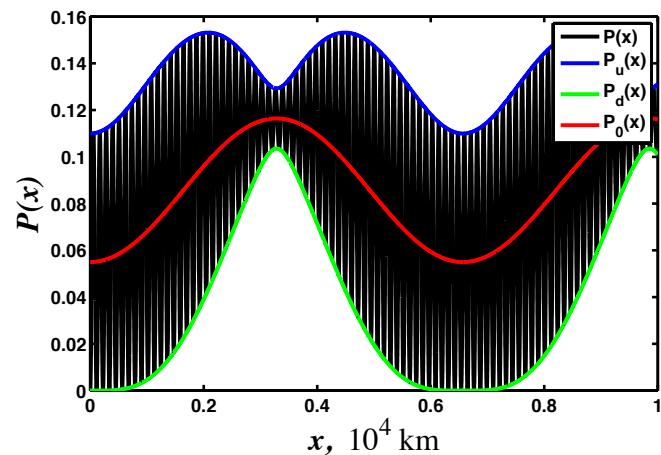
- Параметры нейтрино:  $E_\nu = 10 \text{ MeV}$ ,  $\Delta m^2 = 8 \times 10^{-5} \text{ eV}^2$ ,  $\theta_{\text{vac}} = 0.6$ . Осцилляции  $\nu_e \rightarrow \nu_\mu$ .
  - Параметры вещества:  $n_e = n_p = n_n$  (изоскалярное вещество),  $\rho = (10 - 200) \text{ g/cm}^3$ . Подобное вещество вполне может существовать в области ударной волны.
  - Рассматривается нейтрино с большим переходным моментом.
  - Параметры магнитной энергии:  $\mu B \sim (10^{-13} - 10^{-12}) \text{ eV}$ . Если  $\mu = 10^{-11} \mu_B$ , то  $B \sim (10^6 - 10^7) \text{ G}$ .
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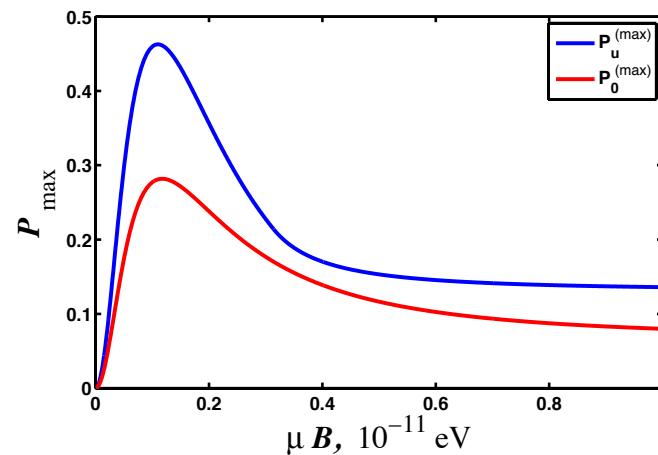
(a)



(b)

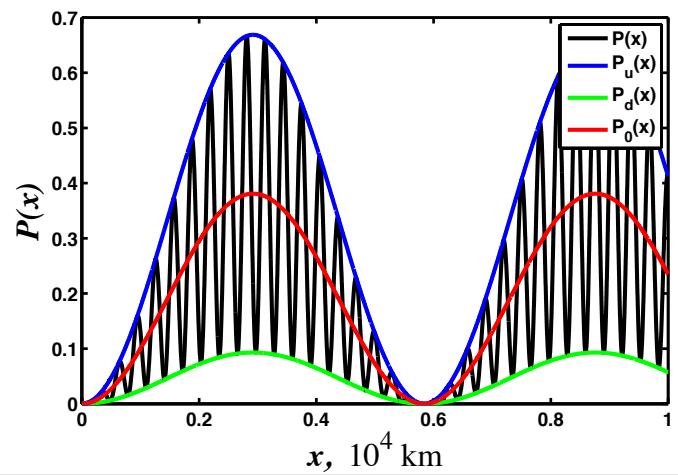


(c)

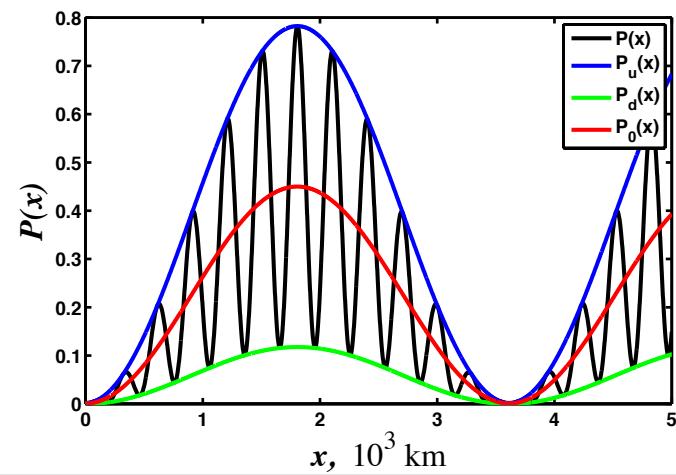


(d)

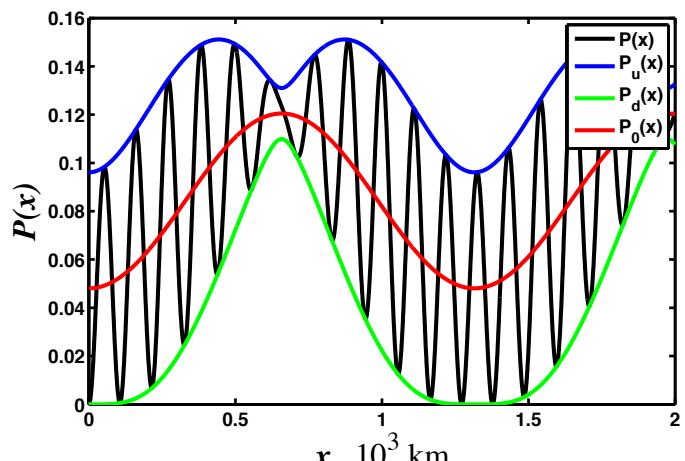
$\rho = 10 \text{ g/cm}^3$ , (a)  $\mu B = 5 \times 10^{-13} \text{ eV}$ , (b)  $\mu B = 1.1 \times 10^{-12} \text{ eV}$ , (c)  $\mu B = 5 \times 10^{-12} \text{ eV}$



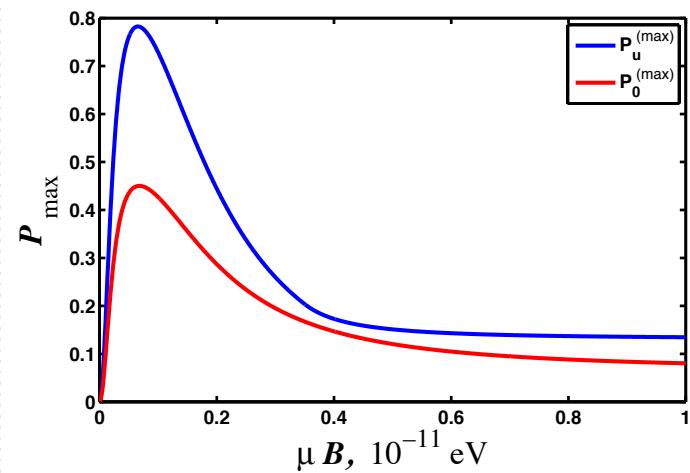
(a)



(b)

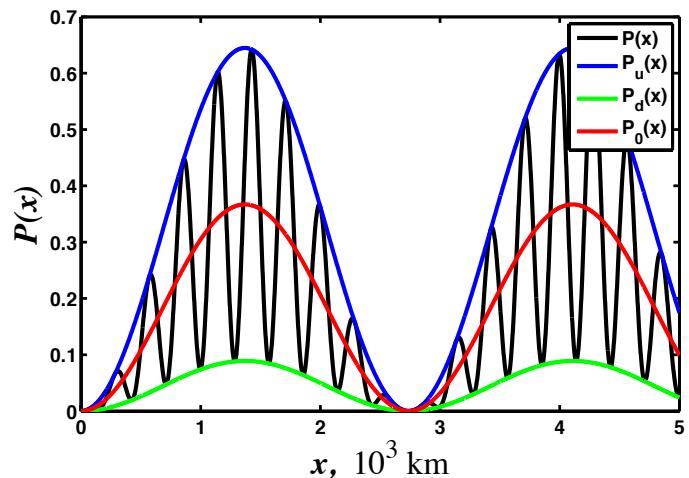


(c)

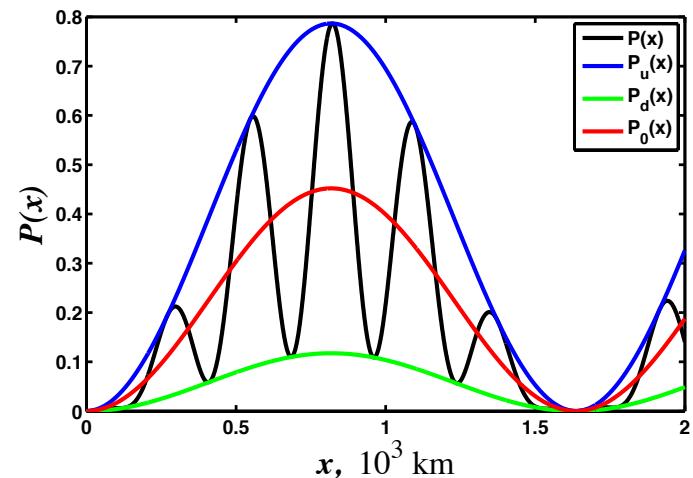


(d)

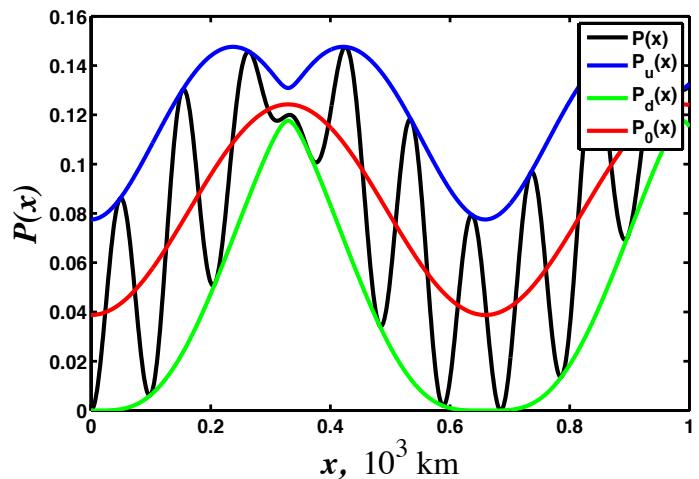
$\rho = 50$  g/cm<sup>3</sup>, (a)  $\mu B = 3.5 \times 10^{-13}$  eV, (b)  $\mu B = 6.6 \times 10^{-13}$  eV, (c)  $\mu B = 5 \times 10^{-12}$  eV



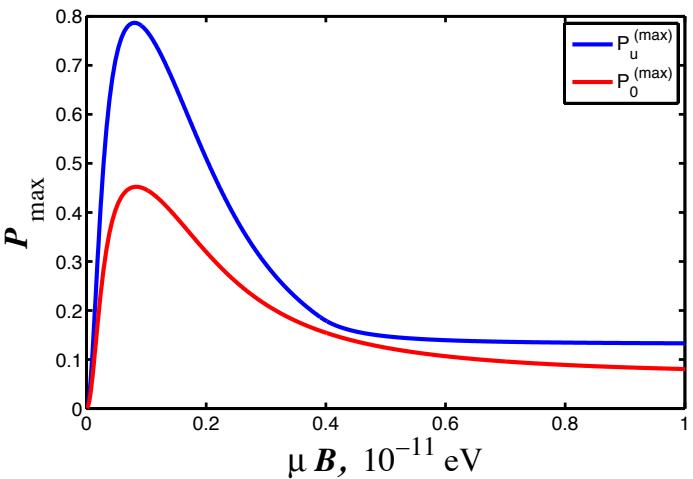
(a)



(b)



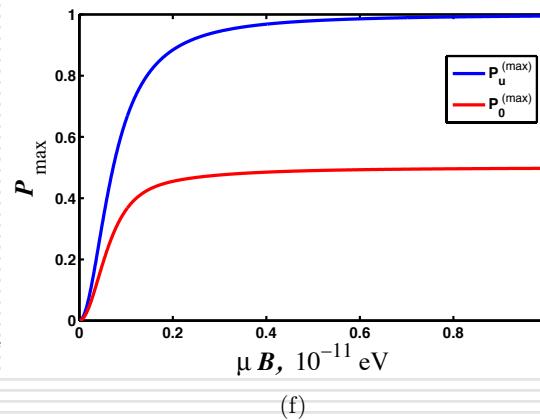
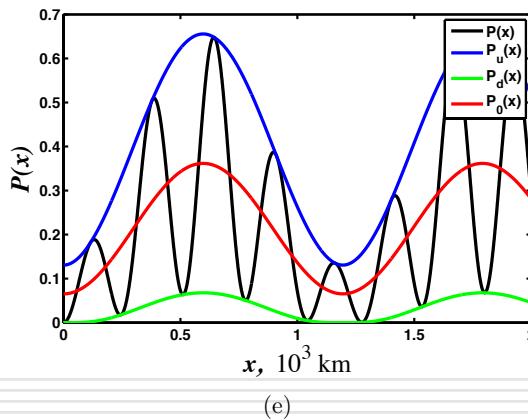
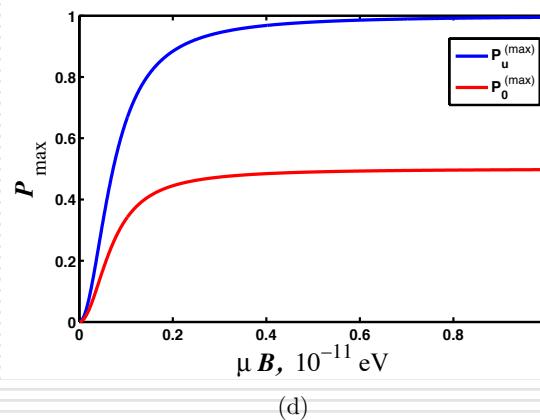
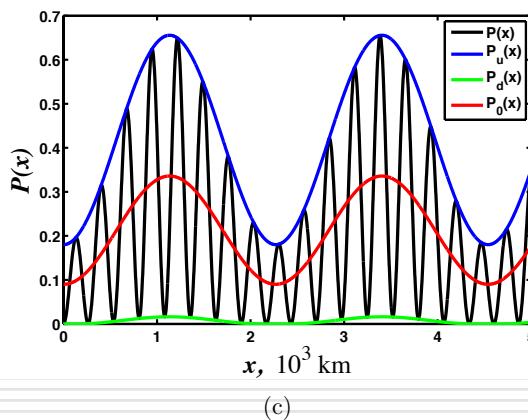
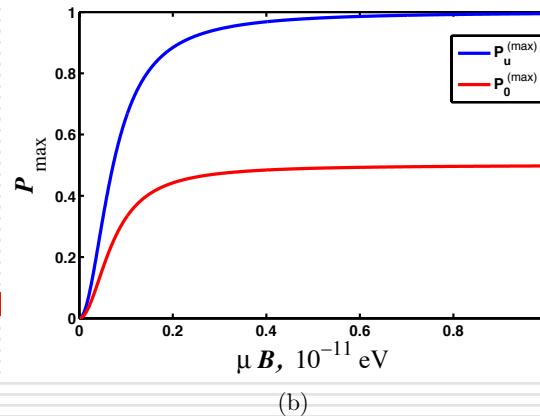
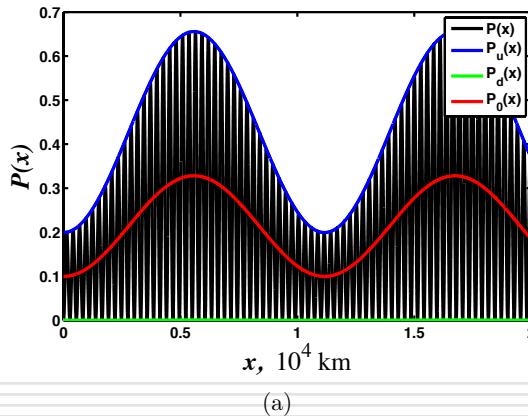
(c)



(d)

$\rho = 100 \text{ g/cm}^3$ , (a)  $\mu B = 4 \times 10^{-13} \text{ eV}$ , (b)  $\mu B = 8 \times 10^{-13} \text{ eV}$ , (c)  $\mu B = 5 \times 10^{-12} \text{ eV}$

# Осцилляции майорановских нейтрино



(a), (c), (e):  
 $\mu B = 10^{-12} \text{ eV}$ ;  
 (a) и (b):  
 $\rho = 20 \text{ g/cm}^3$ ;  
 (c) и (d):  
 $\rho = 100 \text{ g/cm}^3$ ;  
 (c) и (d):  
 $\rho = 200 \text{ g/cm}^3$

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# **ОСЦИЛЛЯЦИИ НЕЙТРИНО В ВЕЩЕСТВЕ В РАМКАХ КВАНТОВОЙ ТЕОРИИ ПОЛЯ**

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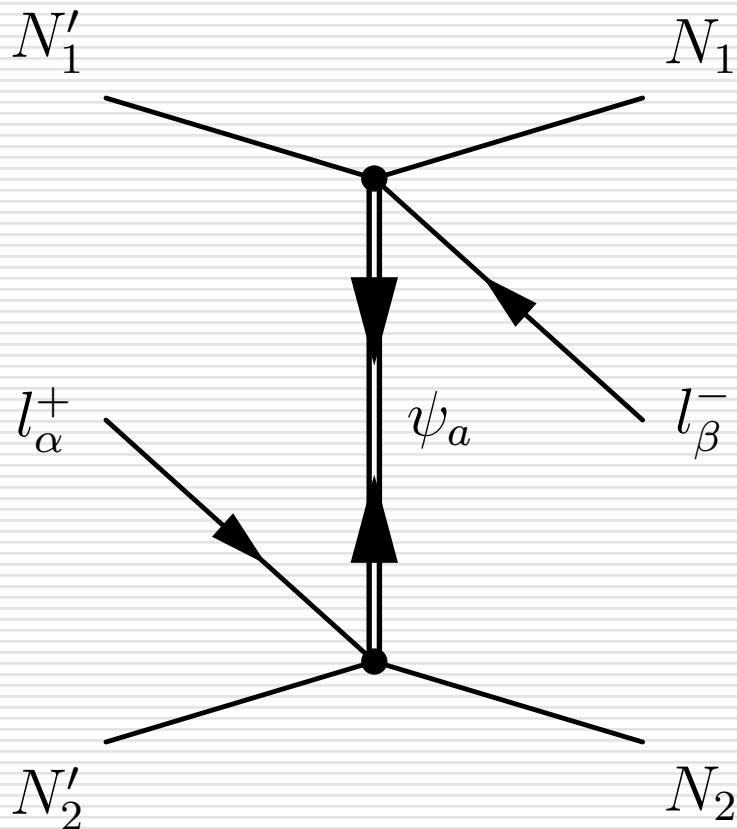
# Майорановские нейтрино в веществе

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- Если нейтрино майорановская частица, то  $\nu = \text{anti-}\nu$ .
  - Возможны переходы  $\nu \rightarrow \text{anti-}\nu$ .
  - Данный тип осцилляций соответствует, редкому и пока не обнаруженному безнейтринному двойному бета-распаду ( $0\nu2\beta$ ):  $(A,Z) \rightarrow (A,Z+2) + 2e^-$ .
  - Поскольку  $(0\nu2\beta)$ -распад происходит в плотном веществе внутри ядра, необходимо исследовать влияние вещества на распространение нейтрино.
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# Амплитуда процесса

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- В данном подходе, основанном на квантовой теории поля, вычисляется показанная фейнмановская диаграмма.
- Точки 1 и 2 являются макроскопически разнесенными в пространстве.
- Нейтрино является виртуальной частицей.
- Если поглощается и излучается лептоны с разным знаком заряда, то виртуальное нейтрино является майорановской частицей.
- Пропагатор нейтрино показан широкой линией поскольку в нем точно учитывается вклад ядерного вещества

# Результат вычисления процесса

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- Рассмотрен случай малого угла смешивания. При этом пропагатор нейтрино диагонален в массовом базисе.
  - Полное сечение процесса подавляется если импульс нейтрино  $p_a = (E_a^2 - m_a^2)^{1/2}$  меньше эффективного потенциала взаимодействия:  $p_a < V_a \sim G_F n_{\text{matter}}$ .
  - Вклад гипотетических массивных майорановских нейтрино в  $(0\nu 2\beta)$ -распад незначительный.
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# Публикации в рецензируемых изданиях по результатам исследований нейтринных осцилляций

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- **M. Dvornikov**, "Spin-flavor oscillations of Dirac neutrinos described by relativistic quantum mechanics", Physics of Atomic Nuclei **75**, 227 – 238 (2012), [arXiv:1008.3115](#).
  - **M. Dvornikov**, "Field theory description of neutrino oscillations", in 'Neutrinos: Properties, Sources and Detection', ed. by J. P. Greene (New York, Nova Science Publishers, Inc., 2011), pp. [23 – 90](#), [arXiv:1011.4300](#).
  - **M. Dvornikov** and J. Maalampi, "Oscillations of Dirac and Majorana neutrinos in matter and magnetic field", Physical Review D **79**, 113015 (2009), [arXiv:0809.0963](#).
  - **M. Dvornikov**, "Evolution of mixed particles interacting with classical sources", Physics of Atomic Nuclei **72**, 116 – 127 (2009), [hep-ph/0610047](#).
  - **M. Dvornikov**, "Neutrino oscillations in matter and in twisting magnetic fields", Journal of Physics G: Nuclear and Particle Physics **35**, 025003 (2008), [arXiv:0708.2328](#).
  - **M. Dvornikov**, "Neutrino spin-flavor oscillations in rapidly varying external fields", Physics of Atomic Nuclei **70**, 342 – 348 (2007), [hep-ph/0410152](#).
  - **M. Dvornikov** and J. Maalampi, "Evolution of mixed Dirac particles interacting with an external magnetic field", Physics Letters B **657**, 217 – 227 (2007), [hep-ph/0701209](#).
  - **M. Dvornikov**, "Evolution of coupled fermions under the influence of an external axial-vector field", The European Physical Journal C: Particles and Fields **47**, 437 – 444 (2006), [hep-ph/0601156](#).
  - **M. Dvornikov**, "Evolution of coupled classical fields", Physics Letters B **610**, 262 – 269 (2005), [hep-ph/0411101](#).
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# Публикации по особенностям квантования майорановских нейтрино во внешних полях

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- **M. Dvornikov** and D. M. Gitman, "*Canonical quantization, path integral representations, and pseudoclassical description of massive Weyl neutrinos in external backgrounds*", Physical Review D **87**, 025027 (2013), [arXiv:1211.5367](https://arxiv.org/abs/1211.5367).
  - **M. Dvornikov**, "*Canonical quantization of a massive Weyl field*", Foundations of Physics **42**, 1469 – 1479 (2012), [arXiv:1106.3303](https://arxiv.org/abs/1106.3303).
  - **M. Dvornikov**, "*Evolution of a dense neutrino gas in matter and electromagnetic field*", Nuclear Physics B **855**, 760 – 773 (2012), [arXiv:1108.5043](https://arxiv.org/abs/1108.5043).
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Глава 2

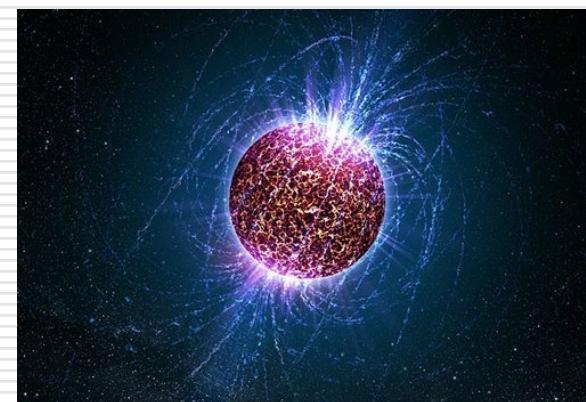
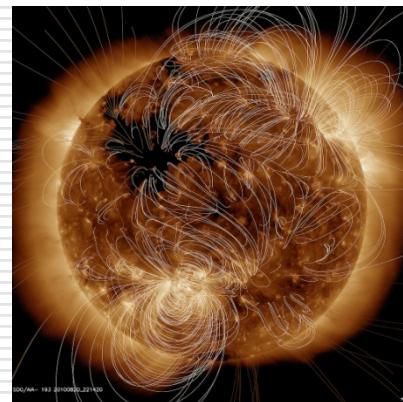
# **СИЛЬНЫЕ МАГНИТНЫЕ И ГИПЕРМАГНИТНЫЕ ПОЛЯ В РАННЕЙ ВСЕЛЕННОЙ**

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# Cosmic magnetic fields: Overview

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(1950's – 1960's) – Alfven, Biermann, Chandrasekhar, Parker etc – first studies on large-scale cosmic magnetic fields



Galactic and intergalactic  
magnetic fields  $\sim 1 \text{ }\mu\text{G}$

Stellar magnetic  
fields  $\sim (1 - 100) \text{ G}$

Magnetic fields in  
compact stars  
(magnetars)  $> 10^{15} \text{ G}$

Recent reviews:

- Durrer & Neronov 2013 *Astron. Astrophys. Rev.* **21** 62–70
- Charbonneau 2010 *Living Rev. Solar Phys.* **7** 3–91
- Mereghetti et al. 2015 *Space Sci. Rev.* **191** 315–338

# Cosmic magnetic field (CMF) of cosmological origin

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- Our universe is permeated by CMF which are dynamo amplified from a seed field.
  - The origin of the seed field is unknown.
  - Seed fields can be produced by MHD mechanisms during epoch of galaxy formation, or ejected by first supernovae or active galactic nuclei.
  - Another scenario suggests that a seed field can originate from much earlier epoch of the Universe expansion: inflation era, phase transitions in radiation era etc.
  - We choose the latter possibility.
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# Strength of CMF

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- Upper bound on CMF can be obtained from Faraday rotation measure:  $B_{\text{CMF}} < 10^{-9} \text{ G}$
  - CMF influence the propagation of intergalactic cosmic rays, e.g. from blazars (compact quasars) to Milky Way
  - Lower bound (Neronov et al., 2010) –  
 $B_{\text{CMF}} > 10^{-16} \text{ G}$  – from non-observation of secondary photons with  $E = 1 \text{ GeV}$  in the initial flux with  $E = 1 \text{ TeV}$ :  $\gamma\gamma \rightarrow e^+e^-$ ;  $e^-\gamma_{\text{CMB}} \rightarrow e^-\gamma(\text{GeV})$
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# **НЕУСТОЙЧИВОСТЬ МАГНИТНОГО ПОЛЯ В НЕЙТРИННОМ ГАЗЕ**

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# Мотивация

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- Нейтрино участвуют в электрослабых взаимодействиях, в которых нарушается пространственная четность.
  - Есть вероятность появления неустойчивости магнитного поля, приводящего к росту затравочного поля.
  - Если не учитывать магнитный момент нейтрино, то эти частицы напрямую не взаимодействуют с электромагнитными полями.
  - Однако, если наряду с нейтрино рассмотреть электрон-позитронную плазму, то можно осуществить взаимодействие между нейтрино и фотонами.
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# Photon dispersion in background matter

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- We consider electron-positron plasma electroweakly interacting with neutrino-antineutrino gas
- The interaction between electrons and neutrinos is parity violating

$$\mathcal{L}_{\text{int}} = \bar{\psi}_e \gamma_\mu \left[ V_L^\mu \left( \frac{1 - \gamma^5}{2} \right) + V_R^\mu \left( \frac{1 + \gamma^5}{2} \right) \right] \psi_e, \quad V_{L,R}^\mu \sim G_F \langle J_{vL,R}^\mu \rangle$$

Photon polarization tensor in background matter with parity violating interaction between particles

$$\Pi_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi_T + e_\mu e_\nu \Pi_L + i \epsilon_{\mu\nu\alpha\beta} k^\alpha \left( V_L^\beta - V_R^\beta \right) \Pi_P$$

- $\Pi_T$  – vacuum polarization in QED
- $\Pi_L$  – plasma frequency or plasmon mass in QED plasma ( $e_\mu k^\mu = 0$ ,  $e^2 = -1$ )
- $\Pi_2 = (V_L^0 - V_R^0)$   $\Pi_P$  – new form factor in parity violating isotropic medium

# Generalized MHD

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Chern-Simons term  $\Pi_2$ :  $J_\mu = \Pi_{\mu\nu} A^\nu$  or  $\mathbf{J}_5 = \Pi_2 \mathbf{B}$

In the presence of the CS term the Maxwell equations are modified

$$i(\mathbf{k} \times \mathbf{B}) = -i\omega \mathbf{E} + \mathbf{j} + \mathbf{j}_5, \quad i(\mathbf{k} \times \mathbf{E}) = i\omega \mathbf{B}, \quad (\mathbf{k} \cdot \mathbf{B}) = 0, \quad \mathbf{j} = \sigma \mathbf{E}, \quad \mathbf{j}_5 = \Pi_2 \mathbf{B}$$

Using the MHD approximation  $\sigma \gg \omega$ , One gets the Faraday equation

$$\frac{\partial \mathbf{B}}{\partial t} = \alpha(\nabla \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \quad \alpha = \frac{\Pi_2}{\sigma}, \quad \Pi_2 \equiv \Pi_2(k_0 = 0), \quad \eta = \frac{1}{\sigma}$$

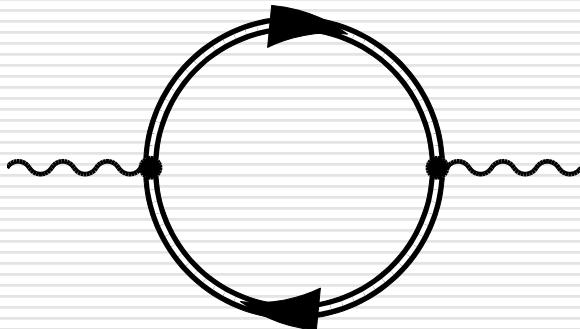
The Faraday equation has the unstable solution

$$B(k, t) = B_0 \exp \left[ \int_{t_0}^t (|\alpha| k - \eta k^2) dt' \right]$$

If  $k < |\alpha|/\eta$ , this solution describes the exponential growth of a seed magnetic field  $B_0$

---

# One loop contribution to polarization tensor

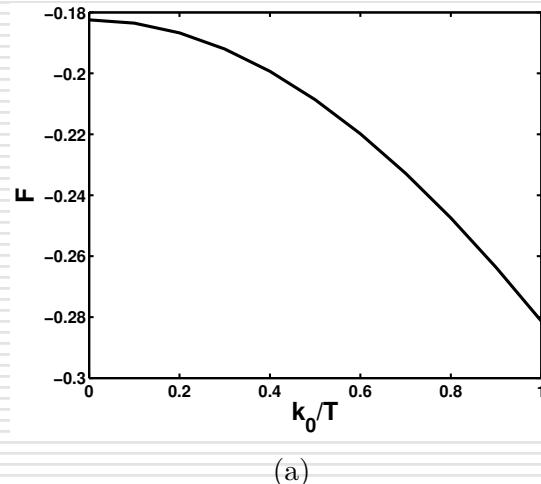


$$\Pi_2 = \frac{\alpha_{em}}{\pi} (V_L^0 - V_R^0) F$$

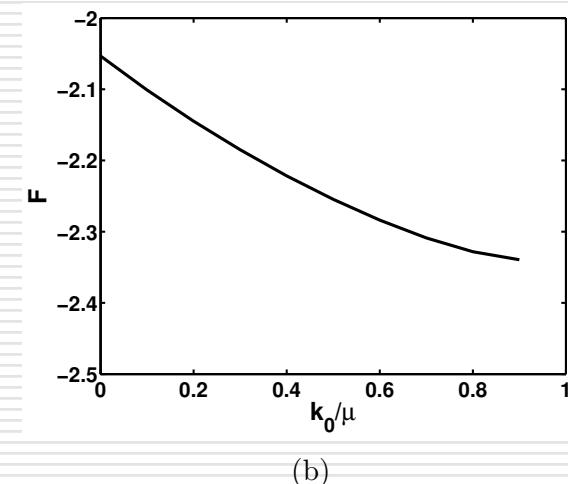
(a) Hot relativistic plasma

(b) Degenerate relativistic plasma

Electron propagators are shown with broad lines since they exactly account for the interaction with neutrinos



(a)



(b)

# Evolution of magnetic fields in the early universe driven by neutrino asymmetry

---

The CS parameter  $\Pi_2$  is nonzero only if there is a neutrino asymmetry  $\Delta n = n(v) - n(\text{anti-}v) \neq 0$  and  $F(k_0 \ll T) \neq 0$ .

It is an open issue in cosmology if there is  $\Delta n \neq 0$

We suppose that CMF are generated in our scenario.

The length scale should be less than the horizon to fulfill the causality condition.

The lower bound on the neutrino asymmetries  $\xi_\nu \sim |\xi_{\nu_e} - \xi_{\nu_e} - \xi_{\nu_e}| > \frac{1.1 \times 10^{-6} \sqrt{g^*/106.75}}{(T/\text{MeV})}$

$$\Lambda_B \sim \frac{\eta}{|\alpha|} < l_H = H^{-1}, \quad \alpha = \frac{\Pi_2}{\sigma}, \quad \sigma \approx 10^2 T$$

$$\xi_\nu = \frac{\mu_\nu}{T} = \frac{6\Delta n_\nu}{T^3}$$

The obtained lower bound is consistent with the well known Big Bang nucleosynthesis constraint (Dolgov et al. (2002); Mangano et al. (2012))

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# **ЭВОЛЮЦИЯ МАГНИТНОГО ПОЛЯ ЗА СЧЕТ КИРАЛЬНОГО МАГНИТНОГО ЭФФЕКТА В ТУРБУЛЕНТНОМ ВЕЩЕСТВЕ**

---

# CME in a nutshell

Helicity is strongly correlated with the momentum for massless particles



While interacting with a constant magnetic field  $\mathbf{B}$ , the spin of a charged particle (e.g. an electron) is aligned opposite  $\mathbf{B}$  and the spin of an antiparticle (a positron) along  $\mathbf{B}$ , at zero Landau level

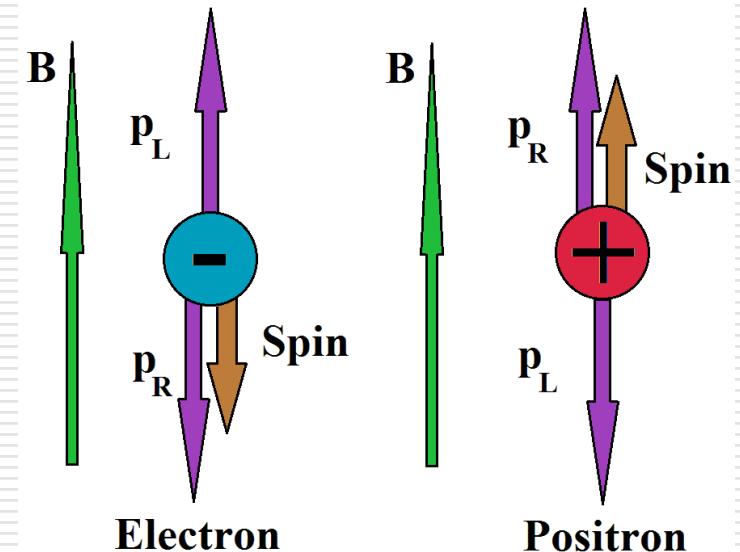
Left electrons move along  $\mathbf{B}$ , whereas right ones oppose  $\mathbf{B}$

Thus we can expect a flux of charged particles, i.e. electric current, along  $\mathbf{B}$

The detailed calculation by Vilenkin (1980) shows that  $\mathbf{J} = \frac{2\alpha_{em}}{\pi} \mu_5 \mathbf{B}$ ,  $\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$

If fermions electroweakly interact with background matter, Dvornikov & Semikoz (2015) found that the electric current has the form

$$\mathbf{J} = \frac{2\alpha_{em}}{\pi} (\mu_5 + V_5) \mathbf{B}, V_5 = \frac{1}{2} (V_L - V_R) \quad V_{L,R} \sim G_F \text{ are the effective potentials for the electroweak interaction of left and right fermions}$$



# Magnetic helicity

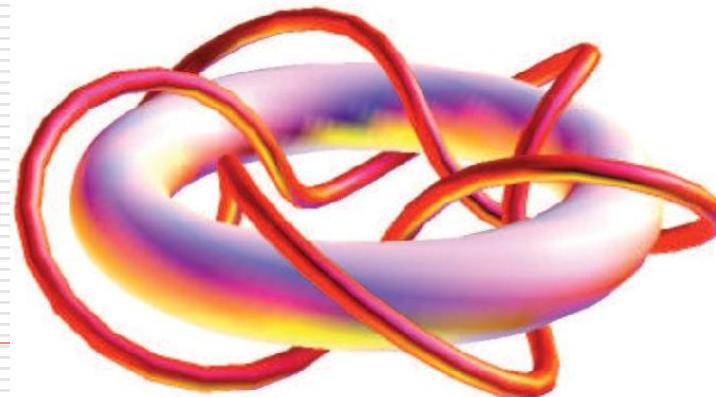
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$$H = \int d^3x (\mathbf{A} \cdot \mathbf{B})$$

- Magnetic helicity was first introduced by Gauss (1833)
- Magnetic helicity is conserved in the perfectly conducting fluid
- Magnetic helicity is gauge invariant
- In the system of two linked magnetic fluxes, magnetic helicity takes the form (Berger, 1999)

$$H = 2L\Phi_1\Phi_2$$

$$L = 0, \pm 1, \pm 2, \dots$$



$$|L| = 5$$

# Relativistic MHD in presence of CME

---

We shall describe the evolution of magnetic field driven by CME in turbulent matter which is a relativistic plasma

Evolution equations for relativistic plasma in one fluid approximation

$$(\partial_t + \mathbf{v} \cdot \nabla) \epsilon + (\epsilon + P) \nabla \cdot \mathbf{v} = 0, \quad (\epsilon + P)(\partial_t + \mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \mathbf{j} \times \mathbf{B} + (\epsilon + P)v \nabla^2 \mathbf{v}$$

Neglecting the displacement current  $d\mathbf{E}/dt$ ,  
that is valid in MHD approximation  $\omega \ll \sigma_{cond}$ ,  $\mathbf{j} = \nabla \times \mathbf{B}$ ,  $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$   
the Maxwell equations take the form

Accounting for the CME  
contribution, the electric  
current reads

$$\mathbf{j} = \mathbf{j}_{Ohm} + \mathbf{j}_{CME} = \sigma_{cond} [\mathbf{E} + (\mathbf{v} \times \mathbf{B})] + \mathbf{j}_{CME}$$

We get the  
modified  
Faraday  
equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta_m \nabla^2 \mathbf{B} + \frac{\Pi}{\sigma_{cond}} (\nabla \times \mathbf{B}), \quad \Pi = \frac{2\alpha_{em}}{\pi} (\mu_5 + V_5)$$

# Model to account for the turbulent motion of matter

---

How to take into account the plasma motion in the generation of magnetic fields?

We shall study ultrarelativistic background matter (hot plasma in the early universe)

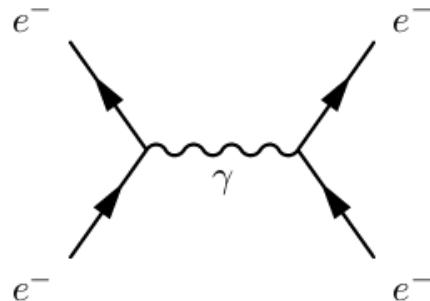
We assume that the Lorentz force in Navier-Stokes equation is dominant

$$\varepsilon + P = \frac{4}{3}\varepsilon$$
$$\frac{\partial \mathbf{v}}{\partial t} = \frac{1}{\varepsilon + P}(\mathbf{j} \times \mathbf{B})$$

This approximation is valid since we study turbulent motion with high Reynolds number  $Re = VL/v \gg 1$  and consider relatively large length scales, i.e. we omit nonlinear terms

We use the drag time approximation. The phenomenological drag time parameter  $\tau_d$  is equal to the time of the Coulomb scattering in plasma

$$\mathbf{v} = \frac{\tau_d}{\varepsilon + P}(\mathbf{j} \times \mathbf{B})$$



We shall assume that  $\tau_d \gg$  Larmor radius. In this case the plasma mean velocity is driven mainly by the Lorentz force.

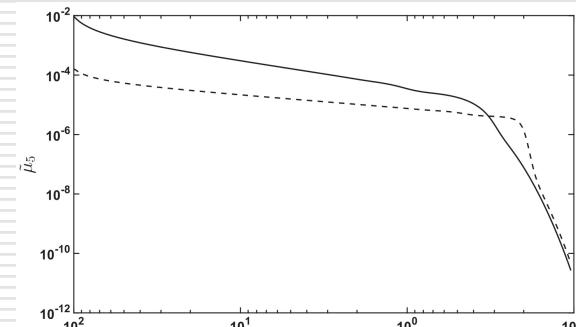
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# Результаты

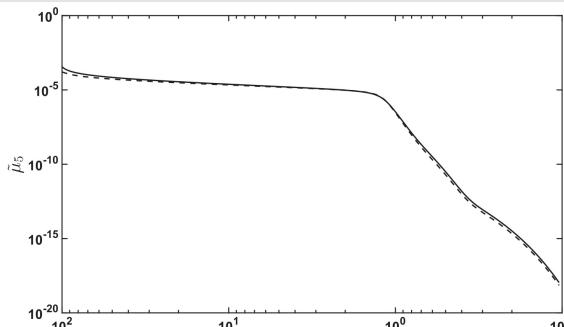
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- Были выведены кинетические уравнения для описания эволюции спектров магнитной спиральности и магнитной энергии учитывающие КМЭ и МГД-турбулентность.
  - Показано, что МГД-турбулентность не может усиливать магнитное поле, в противоположность Campanelli (2007), Sigl et al. (2017).
  - Проанализирована эволюция магнитного поля в турбулентной плазме в ранней вселенной после ЭСФП.
-

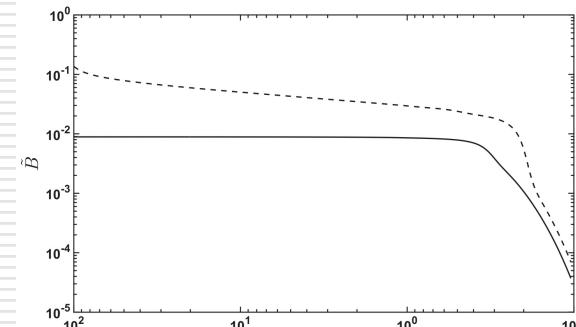
# Results of numerical simulations



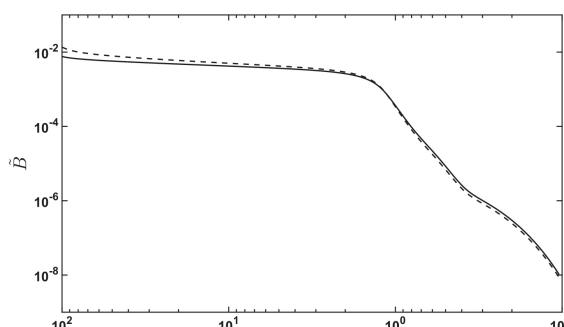
(a)



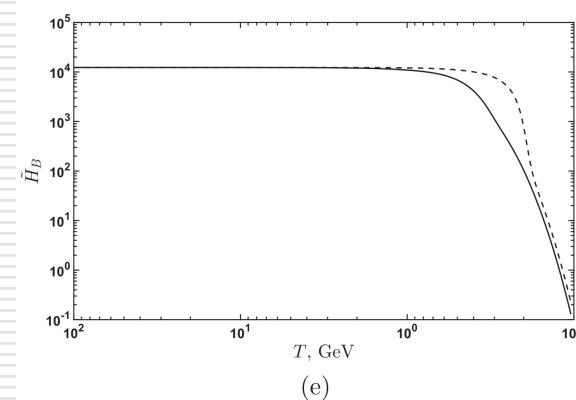
(b)



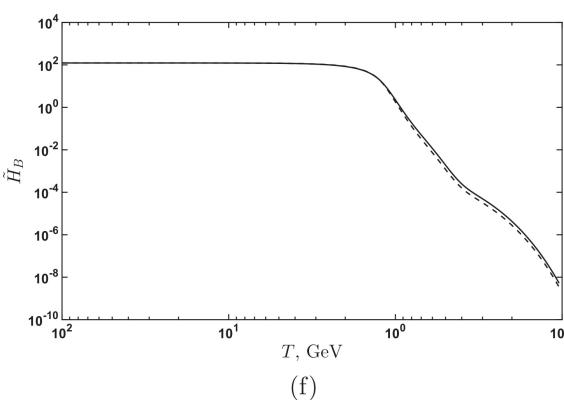
(c)



(d)



(e)



(f)

Dashed lines – only CME  
is accounted for  
(Boyarsky et al., 2012);  
Solid lines – both CME  
and turbulence are  
taken into account

(a), (c) and  
(e) – for  $\tilde{B}_0 = 10^{-1}$

(b), (d) and  
(f) – for  $\tilde{B}_0 = 10^{-2}$   
Note that  $\tilde{B}_0 = 10^{-1}$

corresponds to  $B_{\text{crit}} = 10^{11}$  G at  $T_{\text{BBN}} = 0.1$  MeV, which is a critical strength for BBN nucleosynthesis  
(Cheng et al., 1994)

# **ГИПЕРМАГНИТНЫЕ ПОЛЯ В РАННЕЙ ВСЕЛЕННОЙ**

# Бозоны в ранней вселенной

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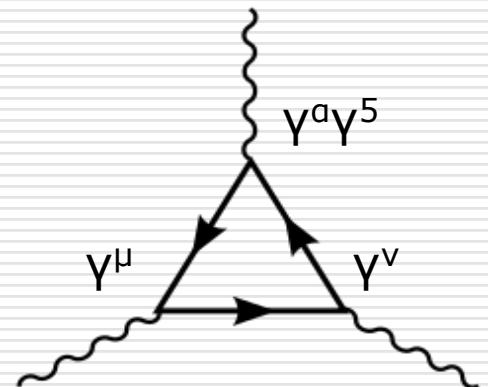
- При температуре  $T > 100 \text{ GeV}$  в плазме ранней вселенной произошел электрослабый кроссовер и симметрия стандартной модели была восстановлена до  $SU(2)_L \times U(1)_Y$ .
  - Вместо электромагнитного поля  $A^\mu$  следует рассматривать гиперзарядовое поле  $Y^\mu = \sin\theta_W Z^\mu + \cos\theta_W A^\mu$ .
  - По аналогии с напряженностями электромагнитного поля можно рассмотреть гиперэлектрическое и гипермагнитное поля  $F_{\mu\nu} \rightarrow Y_{\mu\nu} = (\mathbf{E}_Y, \mathbf{B}_Y)$ .
-

# Абелева аномалия для гиперзарядовых полей

---

- Элементарные частицы становятся эффективно безмассовыми.
- Токи безмассовых частиц не сохраняются из-за квантовой абелевой аномалии:

$$\partial_\mu j_{R,L}^\mu = \pm \frac{g'^2 Y_{R,L}^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \neq 0$$



# Условие равновесия в плазме ранней вселенной

---

- Giovannini & Shaposhnikov (1998) предположили, что равновесное состояние характеризуется:
  - Тремя химическими потенциалами соответствующими законам сохранения 'т Хоофта  $B/3 - L_i = \text{const}$ , для  $i = e, \mu, t$ ;
  - Нулевым средним гиперзарядом (по аналогии с нулевым электрическим зарядом = электронейтральность вселенной);
  - Химическим потенциалом правых лептонов (электронов), поскольку они являются синглетом в стандартной модели.
- Однако, из-за того, что гиперзарядовое поле влияет как на левые так и на правые лептоны за счет абелевой аномалии, необходимо учесть левые частицы при описании кинетики химических потенциалов.

# Новая модель эволюции асимметрий фермионов в присутствии гипермагнитного поля

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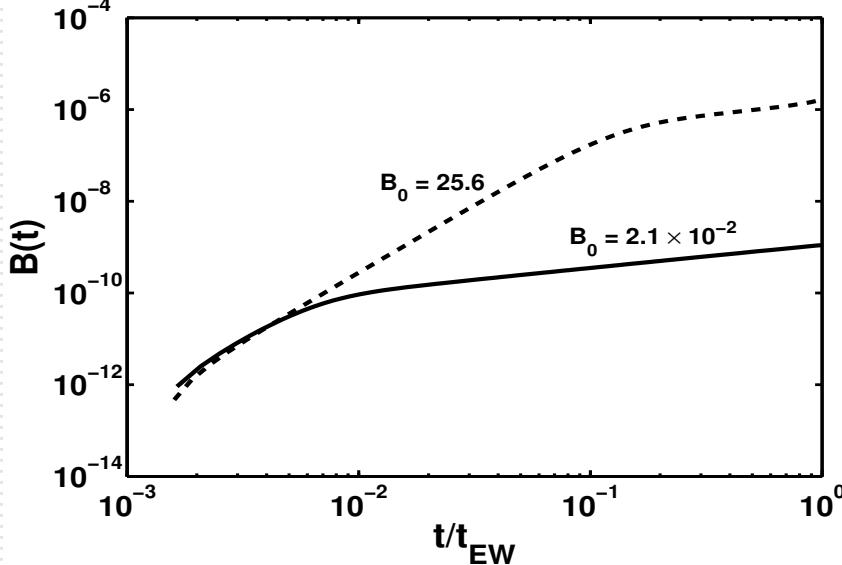
- Рассмотрено самое легкое лептонное поколение (электроны и электронные нейтрино), поскольку распады хиггсовского бозона на левые и правые лептоны  $\Gamma_\phi \sim h_e T$  приходят в равновесие с расширением вселенной  $H \sim T^2$  при самой низкой температуре, т.е. для них остается больше времени для генерации асимметрии левых частиц.
  - Помимо кинетики правых электронов, учтена кинетика левых частиц (электронов и нейтрино).
  - Поскольку левые и правые лептоны могут взаимодействовать с хиггсовским полем, его динамика также учтена.
  - Левые лептоны подвержены сфалеронным переходам, действующим до кроссовера, которые уменьшают левое лептонное число.
  - Необходимость исследования асимметрий лептонов на этой стадии эволюции вселенной мотивирована тем, что различные модели лептогенезиса используют величины асимметрий на момент электрослабого кроссовера (Boyarsky et al., 2012).
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# Результаты

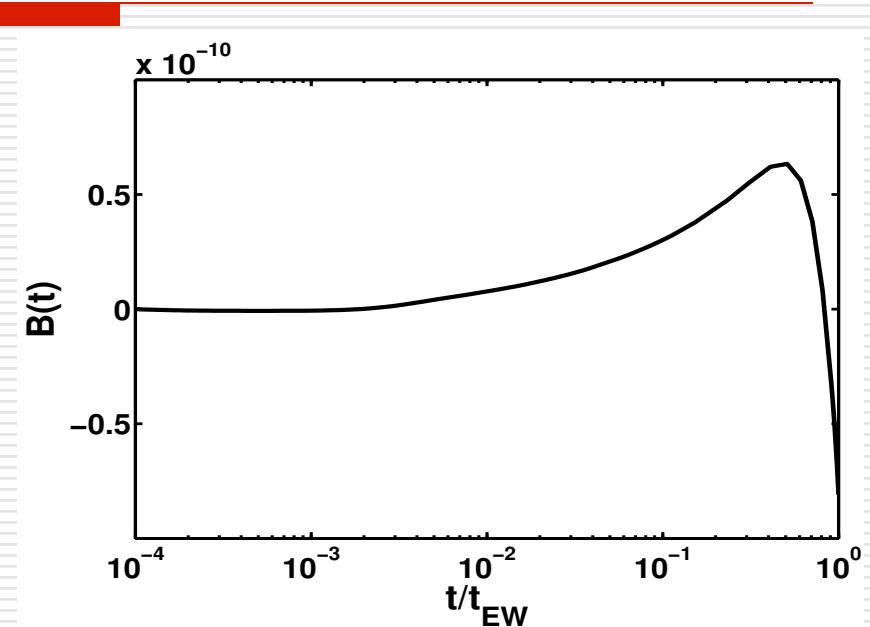
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- Был изучен лептогенезис в сильных гипермагнитных полях до ЭСФП.
  - Используя закон сохранения 'т Хоофта ( $B - L = \text{const}$ ) на этой основе был исследован бариогенезис.
  - Учет левых лептонов не оказывает сильного влияния на генерируемую барионную асимметрию к моменту ЭСФП.
  - Сфалеронные переходы не успевают разрушить барионную асимметрию.
-

# Эволюция барионной асимметрии



(a)



(b)

$B_0 \sim$  обратному масштабу поля  $1/k_0$ , затравочное поле  $B_Y^{(0)} = 10^{19}$  Г,  $t_{EW}$  соответствует времени ЭСФП. (b)  $B_0 = 2 \times 10^{-3}$ .

Можно указать такой масштаб затравочного гипермагнитного поля  $B_0 \sim 10^{-2}$ , при котором вычисленная барионная асимметрия оказывается порядка наблюдаемой  $\sim 10^{-10}$ .

# Публикации в рецензируемых изданиях по результатам исследований

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- **M. Dvornikov** and V. B. Semikoz, "*Influence of the turbulent motion on the chiral magnetic effect in the early universe*", Physical Review D **95**, 043538 (2017), [arXiv:1612.05897](https://arxiv.org/abs/1612.05897).
  - **M. Dvornikov** and V. B. Semikoz, "*Instability of magnetic fields in electroweak plasma driven by neutrino asymmetries*", Journal of Cosmology and Astroparticle Physics 05 (2014) 002, [arXiv:1311.5267](https://arxiv.org/abs/1311.5267).
  - **M. Dvornikov** and V. B. Semikoz, "*Lepton asymmetry growth in the symmetric phase of an electroweak plasma with hypermagnetic fields versus its washing out by sphalerons*", Physical Review D **87**, 025023 (2013), [arXiv:1212.1416](https://arxiv.org/abs/1212.1416).
  - **M. Dvornikov** and V. B. Semikoz, "*Leptogenesis via hypermagnetic fields and baryon asymmetry*", Journal of Cosmology and Astroparticle Physics 02 (2012) 040, [arXiv:1111.6876](https://arxiv.org/abs/1111.6876).
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Глава 3

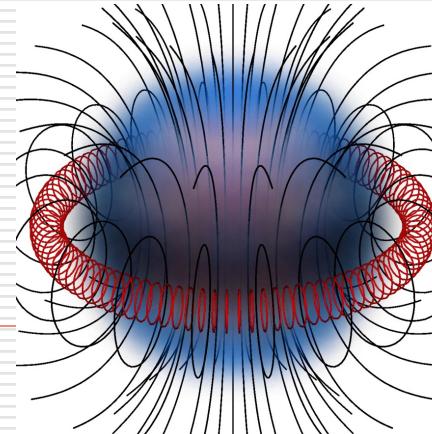
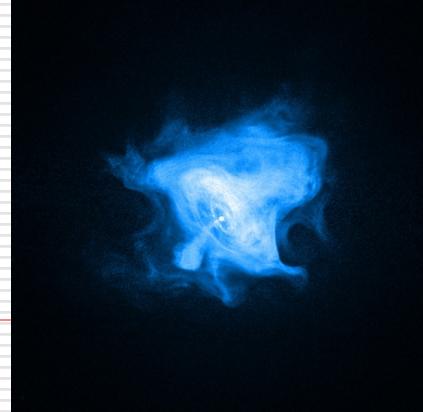
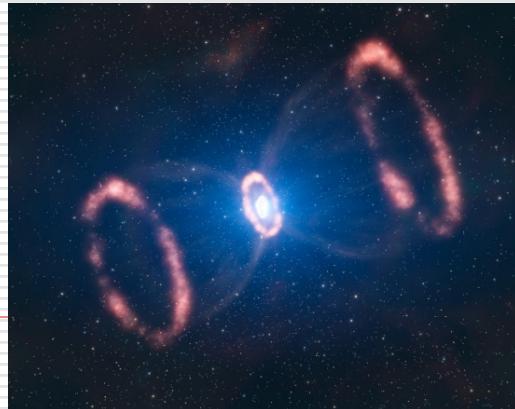
# **МАГНИТНЫЕ ПОЛЯ В КОМПАКТНЫХ ЗВЕЗДАХ**

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# Compact stars and their magnetic fields

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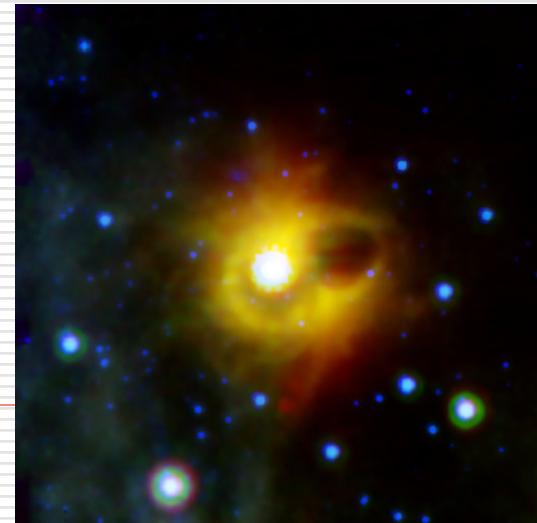
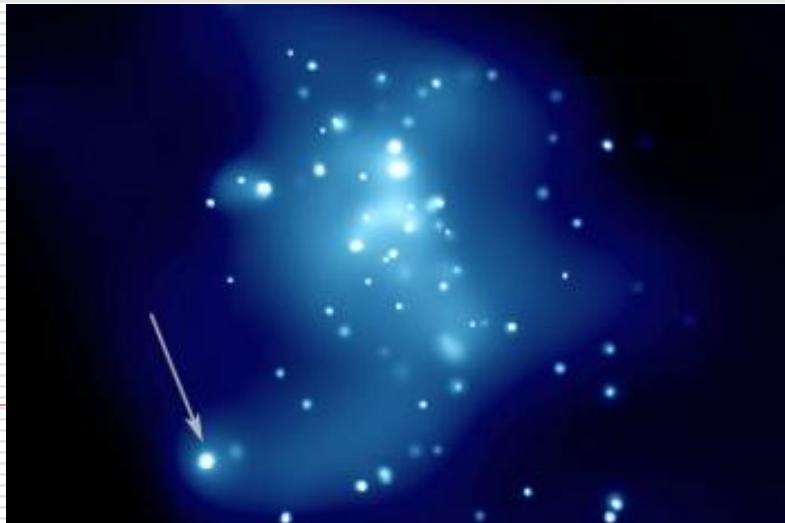
- There is a possibility for the formation of quark matter in compact stars.
- Quark matter can exist in the core of a neutron star. In this case the object is called a hybrid star (HS).
- If the strange matter hypothesis by Witten (1984) is valid, the existence of a star entirely composed of quark matter is possible. It is called a quark star (QS) or a strange star.
- HS/QS has the following characteristics:  $R_{\text{STAR}} = 10 \text{ km}$ ,  $M_{\text{STAR}} = (1.4 - 2)M_{\odot}$ ,  $n = 10^{38} \text{ cm}^{-3}$ . It consists mainly of  $u$  and  $d$  quarks with some admixture of  $s$  quarks.
- Compact star can possess rather strong magnetic fields, ranging from  $10^8 \text{ G}$ , for old pulsars, to  $10^{12} \text{ G}$  for young pulsars.
- The origin of such a strong magnetic field  $B_0 = 10^{12} \text{ G}$  can be explained by the magnetic flux conservation:  $B_0 = B_{\text{PROTO}}(R_{\text{PROTO}}/R_{\text{STAR}})^2 = 10^{12} \text{ G}$ , where  $B_{\text{PROTO}} = (1-10^2) \text{ G}$ ,  $R_{\text{PROTO}} \sim R_{\odot} = 7 \times 10^{10} \text{ cm}$ ,  $R_{\text{STAR}} = 10 \text{ km}$ .



# Magnetars

---

- Anomalous X-ray pulsars and soft gamma-ray repeaters, discovered by Mazets et al. (1979) and Fahlman & Gregory (1981), are supposed to be highly magnetized compact stars with  $B > 10^{15}$  G.
- There are several tens of confirmed known magnetars (Mereghetti et al., 2015).
- Woods (2008) showed that number of magnetars should be comparable with that of pulsars.
- Boldin & Popov (2012) claim that there are hidden magnetars, i.e. strong magnetic field exists inside compact stars.
- There are numerous models, such as the turbulent  $a\text{-}\Omega$  dynamo (Duncan & Thompson, 1992), the strong fossil field (Vink & Kuiper, 2006) etc., how to amplify the magnetic field from  $B_0 = 10^{12}$  G to  $B = 10^{15}$  G.
- Nevertheless the origin of strong magnetic fields in magnetars is still unknown.

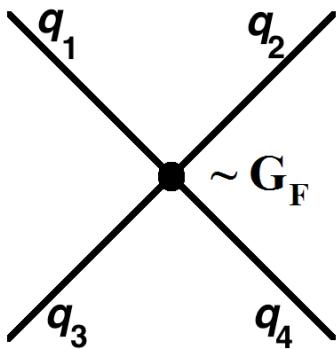


# Features of our model for the generation of magnetic fields in magnetars

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- Quark matter in HS/QS is highly degenerate:  $\mu_u = 239$  MeV and  $\mu_d = 301$  MeV
  - Buballa & Carignano (2016) predict that the chiral phase transition can happen in HS/QS. It means that light  $u$  and  $d$  quarks become effectively massless.
  - Vilenkin (1980) & Dvornikov (2016) showed that CME can take place only when the chiral symmetry is restored.
  - Quarks can interact between themselves by the electroweak forces which violate the parity.
  - We find the correction to the CME owing to the electroweak interaction of quarks.
  - This correction leads to the magnetic field instability, which, in its turn, results in the field growth.
  - One can expect the growth of a seed field  $B_0 = 10^{12}$  G, typical for young pulsars, to the strengths predicted in magnetars.
-

# Electroweak interaction of quarks



We shall describe the electroweak interaction between quarks in the HS/QS matter using the the Fermi approximation.

The amplitude of the  $qq$  forward scattering involves the Fermi constant  $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ .

We suppose that HS/QS consists of  $u$  and  $d$  quarks. The contribution of  $s$  quarks, which are massive, can be neglected.

The effective Lagrangian of the interaction between macroscopically nonmoving and unpolarized  $u$  and  $d$  quarks reads

$$\mathcal{L} = - \sum_{q=u,d} \bar{q} \left( \gamma_0^L V_{qL} + \gamma_0^R V_{qR} \right) q, \quad \gamma_0^{L,R} = \frac{\gamma_0}{2} (1 \mp \gamma^5), \quad \xi = \sin^2 \theta_W \approx 0.23$$

$$V_{uL} = -\frac{G_F}{\sqrt{2}} n_d \left( 1 - \frac{8}{3} \xi + \frac{16}{9} \xi^2 - 2 |V_{ud}|^2 \right), \quad V_{uR} = \frac{G_F}{\sqrt{2}} n_d \left( \frac{4}{3} \xi - \frac{16}{9} \xi^2 \right),$$

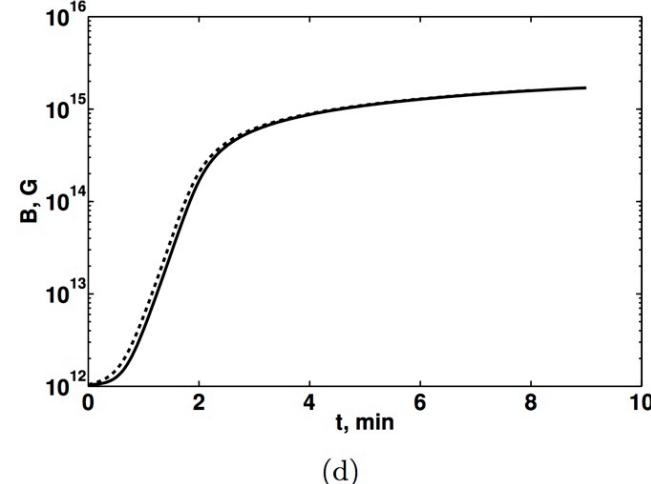
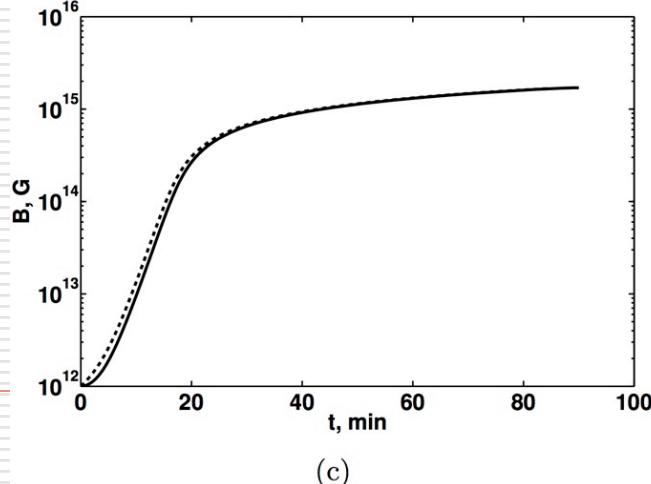
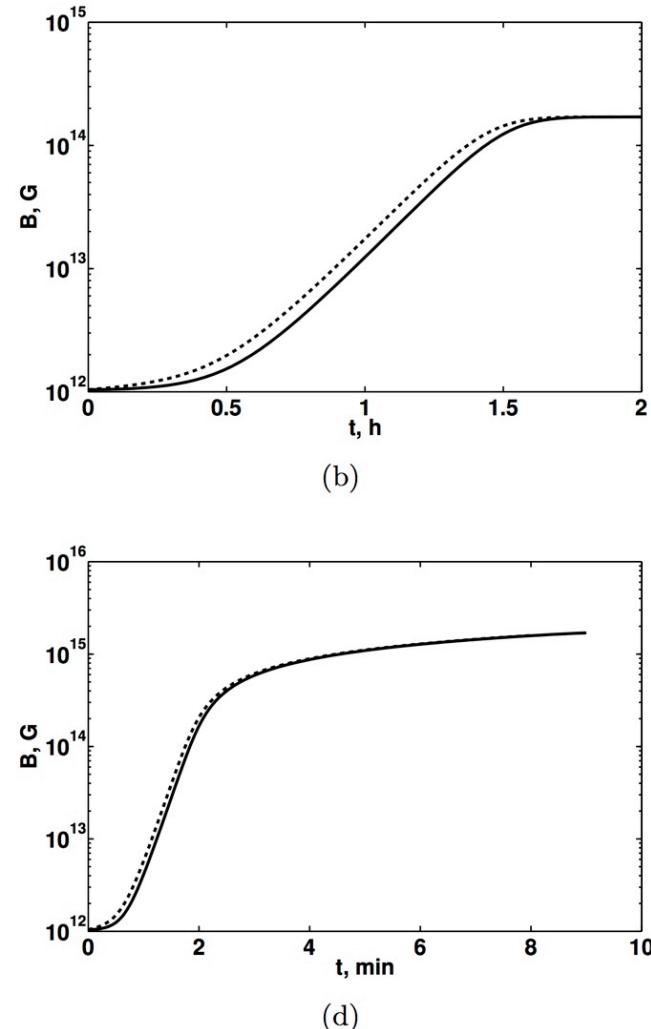
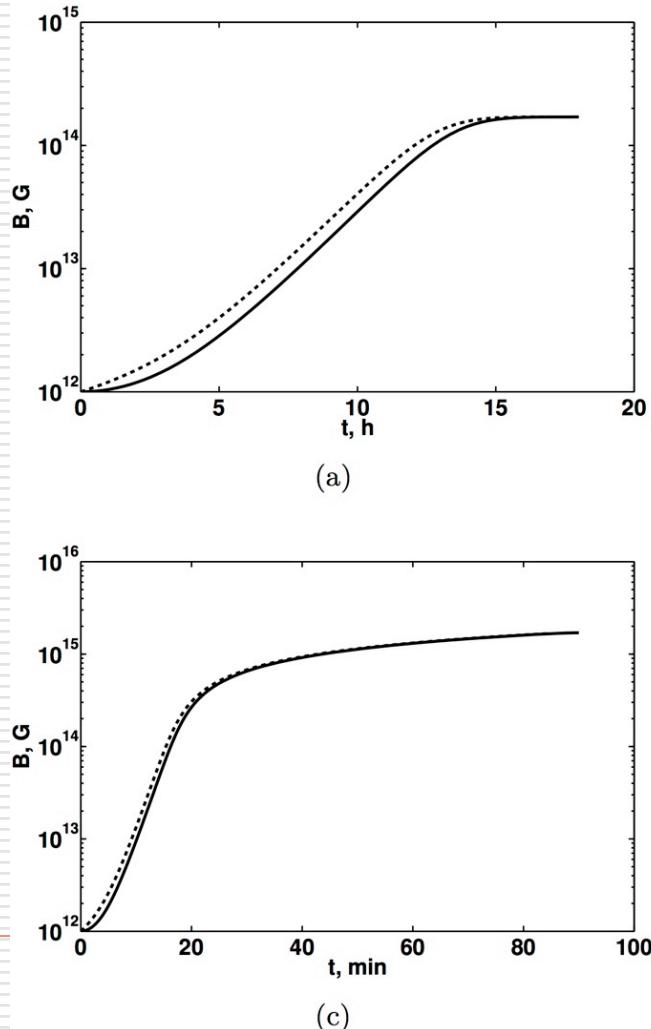
$$V_{dL} = -\frac{G_F}{\sqrt{2}} n_u \left( 1 - \frac{10}{3} \xi + \frac{16}{9} \xi^2 - 2 |V_{ud}|^2 \right), \quad V_{dR} = \frac{G_F}{\sqrt{2}} n_u \left( \frac{2}{3} \xi - \frac{16}{9} \xi^2 \right),$$

Here  $V_{ud} = 0.97$  is the element of the Cabibbo- Kobayashi-Maskawa matrix.

# Evolution of a large-scale magnetic field driven by the CME and electroweak interaction between quarks

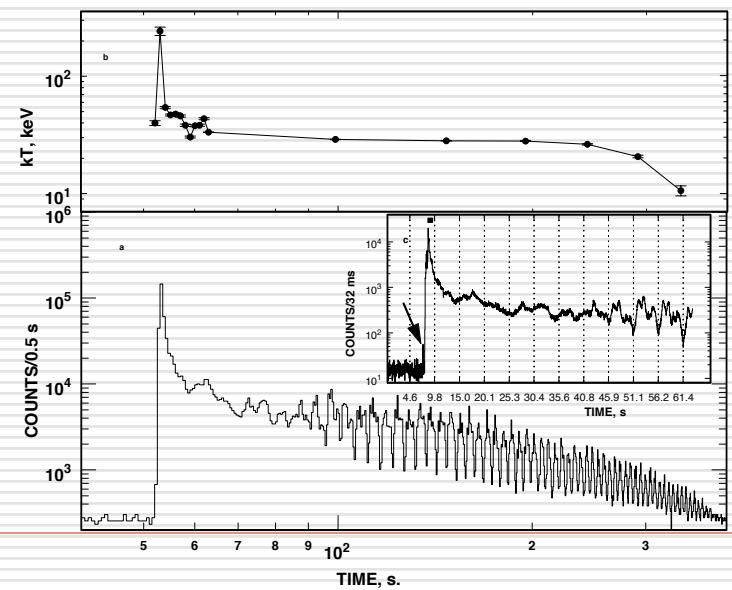
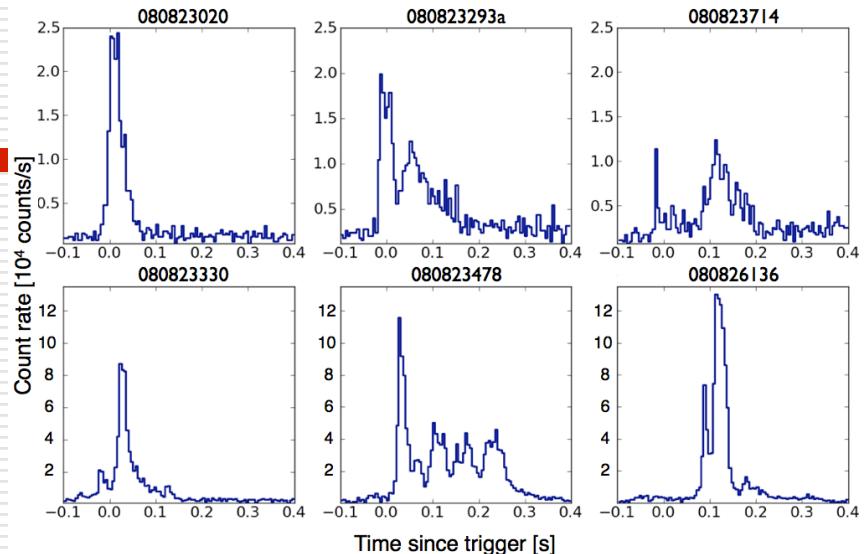
- (a)  $\Lambda_B = 1 \text{ km}$ ,  
 $T_0 = 10^8 \text{ K}$
- (b)  $\Lambda_B = 10^2 \text{ m}$ ,  
 $T_0 = 10^8 \text{ K}$
- (c)  $\Lambda_B = 1 \text{ km}$ ,  
 $T_0 = 10^9 \text{ K}$
- (d)  $\Lambda_B = 10^2 \text{ m}$ ,  
 $T_0 = 10^9 \text{ K}$

Solid lines: zero initial helicity.  
Dashed lines: maximal initial helicity



# Magnetar bursts

- Magnetars are observed by electromagnetic emission in X-ray and gamma-ray regions ranging from short bursts to giant flares
- Magnetar flashes are caused by a twist of magnetic lines in the compact star magnetosphere
- This twist should be associated with a motion of a stellar crust
- Beloborodov & Levin (2014) suggested that plastic deformation of a crust is driven by a thermoplastic wave (TPW)
- Lander (2016) found that TPW can be excited if  $B > 10^{13}$  G
- Li et al. (2016) obtained that, to generate a magnetar flare, the initial magnetic field fluctuation for TPW should have  $\Lambda_B = \text{several meters}$
- The physical process which triggers TPW is unknown
- One can expect that small scale magnetic fields become important to excite TPW



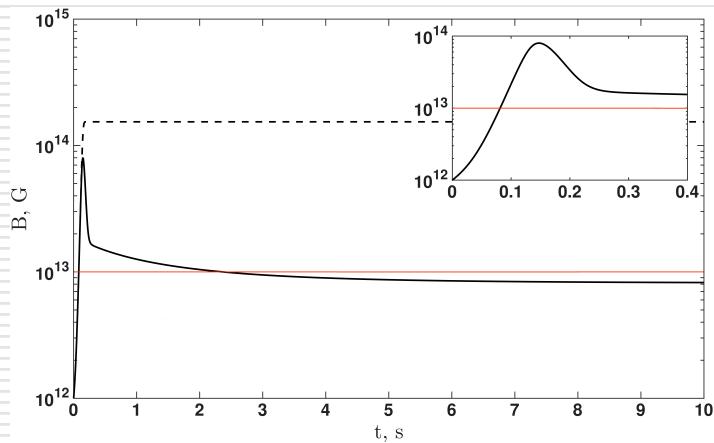
# Модель вспышек магнитаров

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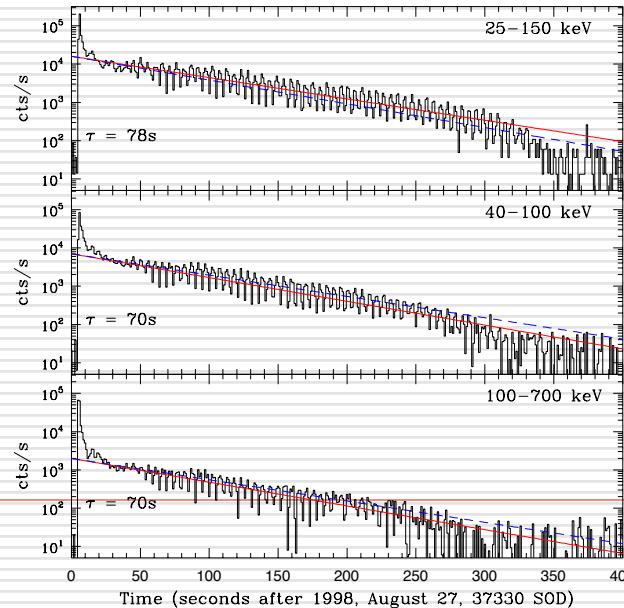
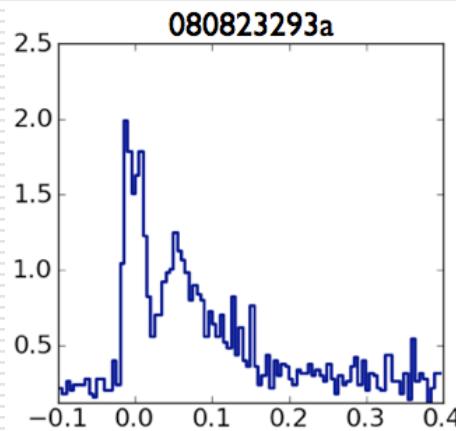
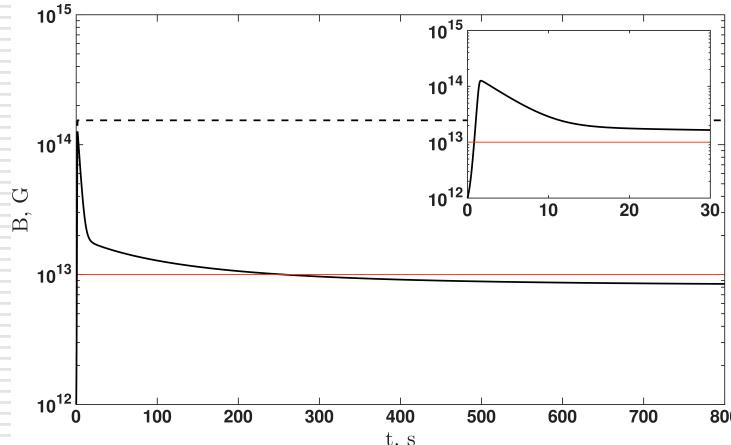
- Поскольку вспышки магнитаров могут генерироваться мелкомасштабным магнитным полем, наряду с КМЭ и электрослабой поправкой, необходимо учесть эффекты турбулентности при генерации подобного поля.
  - Флуктуация магнитного поля с требуемыми характеристиками, возникая во внешнем ядре звезды, где киральная симметрия ненарушенная, возбуждает ТПВ.
  - Эта ТПВ распространяется сквозь кору магнитара, и достигнув поверхности звезды генерирует вспышку.
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# Small scale magnetic field fluctuation and magnetar flashes

□  $T_0 = 10^8 \text{ K}$ ,  $1 \text{ cm} < \Lambda_B < 10 \text{ cm}$



□  $T_0 = 10^8 \text{ K}$ ,  $10 \text{ cm} < \Lambda_B < 1 \text{ m}$



Short burst of magnetar  
SGR J0501+4516  
recorded by Fermi/GBM;  
Huppenkothen et al.  
(2013)

The combined analysis of the  
giant flare of  
magnetar SGR  
1900+14 on August  
27 1998 recorded  
by BeppoSAX  
and Ulysses;  
Feroci et al. (2001)

# Magnetic fields in magnetars driven by neutrino asymmetries

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During  $\sim 1$  ms after the onset of the SN explosion, the regions outside the neutrinosphere deleptonize. The lepton number is carried away by  $v_e$ , leading to the electron neutrino burst with  $L_\nu = 10^{52}$  erg/s. Thus  $\Delta n_\nu \neq 0$ .

Electrons in protoneutron (PNS) star are degenerate and ultrarelativistic. There is a nonzero CS parameter  $\Pi_2$ . We can expect that there is a magnetic field instability driven by  $\Delta n_\nu \neq 0$ .

The magnetic diffusion time is

$$t_{\text{diff}} = \frac{\sigma}{\Pi_2^2} \approx 2.3 \times 10^{-2} \text{ s} \gg t_{\text{burst}} \sim 10^{-3} \text{ s}, \quad \sigma = \frac{1.6 \times 10^{28}}{(T / 10^8 \text{ K})} \left( \frac{n_e}{10^{36} \text{ cm}^{-3}} \right)^{3/2} \text{ s}^{-1}, \quad \Pi_2 = \frac{\sqrt{2}}{\pi} \alpha_{em} G_F n_{v_e} F(0)$$

The disadvantage of the model for magnetic field growth driven by the neutrino asymmetry is that the magnetic field scale is small  $\Lambda_B \sim 10^{-3}$  cm. However, at the subsequent moments of the PNS evolution, when  $\Delta n_\nu = 5 \times 10^{27} \text{ cm}^{-3}$ ,  $\Lambda_B$  can be  $\sim 10$  km

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# Публикации в рецензируемых изданиях по результатам исследований

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  - **M. Dvornikov**, "Relaxation of the chiral chemical potential in dense matter of a neutron star", Russian Physics Journal **59**, 1881 – 1890 (2017), [arXiv:1702.05737](https://arxiv.org/abs/1702.05737).
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  - **M. Dvornikov** and V. B. Semikoz, "Generation of the magnetic helicity in a neutron star driven by the electroweak electron-nucleon interaction", Journal of Cosmology and Astroparticle Physics 05 (2015) 032, [arXiv:1503.04162](https://arxiv.org/abs/1503.04162).
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  - **M. Dvornikov** and V. B. Semikoz, "Magnetic field instability in a neutron star driven by the electroweak electron-nucleon interaction versus the chiral magnetic effect", Physical Review D **91**, 061301 (2015), [arXiv:1410.6676](https://arxiv.org/abs/1410.6676).
  - **M. Dvornikov**, "Impossibility of the strong magnetic fields generation in an electron-positron plasma", Physical Review D **90**, 041702 (2014), [arXiv:1405.3059](https://arxiv.org/abs/1405.3059).
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# Общие результаты

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- Результаты работы опубликованы в 30 статьях в рецензируемых научных изданиях, рекомендованных ВАК
  - По результатам работы сделаны доклады на 24 международных конференциях и многочисленных научных семинарах в России и за рубежом.
  - Всего имеется 63 публикации в рецензируемых научных изданиях.
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